

Mathematica 11.3 Integration Test Results

Test results for the 88 problems in "4.2.1.2 (g sin)^p (a+b cos)^m.m"

Problem 58: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \sin [c + d x])^{11/2}}{a + b \cos [c + d x]} dx$$

Optimal (type 4, 544 leaves, 15 steps):

$$\frac{(-a^2 + b^2)^{9/4} e^{11/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin [c + d x]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right] + (-a^2 + b^2)^{9/4} e^{11/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin [c + d x]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{b^{11/2} d} + \frac{2 a (21 a^4 - 49 a^2 b^2 + 33 b^4) e^6 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\sin [c + d x]}}{21 b^6 d \sqrt{e \sin [c + d x]}} - \frac{a (a^2 - b^2)^3 e^6 \operatorname{EllipticPi}\left[\frac{-2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\sin [c + d x]}}{b^6 (a^2 - b (b - \sqrt{-a^2 + b^2})) d \sqrt{e \sin [c + d x]}} - \frac{a (a^2 - b^2)^3 e^6 \operatorname{EllipticPi}\left[\frac{-2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\sin [c + d x]}}{b^6 (a^2 - b (b + \sqrt{-a^2 + b^2})) d \sqrt{e \sin [c + d x]}} + \frac{2 e^5 (21 (a^2 - b^2)^2 - a b (7 a^2 - 12 b^2) \cos [c + d x]) \sqrt{e \sin [c + d x]}}{21 b^5 d} + \frac{2 e^3 (7 (a^2 - b^2) - 5 a b \cos [c + d x]) (e \sin [c + d x])^{5/2}}{35 b^3 d} - \frac{2 e (e \sin [c + d x])^{9/2}}{9 b d}$$

Result (type 6, 2235 leaves):

$$\frac{1}{d} \left(\frac{a (28 a^2 - 51 b^2) \cos [c + d x]}{42 b^4} + \frac{(-9 a^2 + 14 b^2) \cos [2 (c + d x)]}{45 b^3} + \frac{a \cos [3 (c + d x)]}{14 b^2} - \frac{\cos [4 (c + d x)]}{36 b} \right) \operatorname{Csc}[c + d x]^5 (e \sin [c + d x])^{11/2} - \frac{1}{1680 b^4 d \sin [c + d x]^{11/2}} (e \sin [c + d x])^{11/2} \left(\frac{1}{(a + b \cos [c + d x]) (1 - \sin [c + d x])^2} \right)$$

$$\begin{aligned}
 & 2 (392 a^3 b - 722 a b^3) \operatorname{Cos}[c + d x]^2 \left(a + b \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) \left(\frac{1}{4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}} \right. \\
 & a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] - \right. \\
 & \quad \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + b \operatorname{Sin}[c + d x]\right] + \\
 & \quad \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + b \operatorname{Sin}[c + d x]\right] \right) + \\
 & \left(5 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \right. \\
 & \quad \left. \sqrt{\operatorname{Sin}[c + d x]} \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) / \\
 & \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] + \right. \right. \\
 & \quad 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] + (a^2 - b^2) \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \right) \operatorname{Sin}[c + d x]^2 \right) \\
 & \quad \left. \left. (a^2 + b^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \right) + \frac{1}{(a + b \operatorname{Cos}[c + d x]) \sqrt{1 - \operatorname{Sin}[c + d x]^2}} \\
 & 2 (-280 a^4 + 636 a^2 b^2 - 721 b^4) \operatorname{Cos}[c + d x] \left(a + b \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) \\
 & \left(-\frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \right. \\
 & \left(2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] + \right. \\
 & \quad \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i b \operatorname{Sin}[c + d x]\right] - \\
 & \quad \left. \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i b \operatorname{Sin}[c + d x]\right] \right) + \\
 & \left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \sqrt{\operatorname{Sin}[c + d x]} \right) / \\
 & \left(\sqrt{1 - \operatorname{Sin}[c + d x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2} \right] \right) \operatorname{Sin}[c + d x]^2 \left. \left. (a^2 + b^2 (-1 + \operatorname{Sin}[c + d x]^2)) \right) \right) \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{(a + b \cos [c + d x]) (1 - 2 \sin [c + d x]^2) \sqrt{1 - \sin [c + d x]^2}} \\
 & (840 a^4 - 1764 a^2 b^2 + 959 b^4) \cos [c + d x] \\
 & \cos [2 (c + d x)] \left(a + b \sqrt{1 - \sin [c + d x]^2} \right) \\
 & \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} - \right. \\
 & \frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2 + b^2) \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right]}{b^{3/2} (-a^2 + b^2)^{3/4}} + \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) \\
 & \left. (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin [c + d x]} + i b \sin [c + d x] \right] - \right. \\
 & \frac{1}{b^{3/2} (-a^2 + b^2)^{3/4}} \left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2 + b^2) \operatorname{Log} \left[\sqrt{-a^2 + b^2} + \right. \\
 & \left. (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin [c + d x]} + i b \sin [c + d x] \right] + \frac{4 \sqrt{\sin [c + d x]}}{b} + \\
 & \left. \left(10 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c + d x]^2, \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \sqrt{\sin [c + d x]} \right) / \right. \\
 & \left(\sqrt{1 - \sin [c + d x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c + d x]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin [c + d x]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin [c + d x]^2, \right. \right. \\
 & \left. \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \right) \sin [c + d x]^2 \left(a^2 + b^2 (-1 + \sin [c + d x]^2) \right) \left. \right) - \\
 & \left(36 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c + d x]^2, \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \sin [c + d x]^{5/2} \right) / \\
 & \left(5 \sqrt{1 - \sin [c + d x]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c + d x]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin [c + d x]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin [c + d x]^2, \right. \right. \\
 & \left. \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \right) \sin [c + d x]^2 \left(a^2 + b^2 (-1 + \sin [c + d x]^2) \right) \left. \right) \left. \right)
 \end{aligned}$$

Problem 59: Result unnecessarily involves higher level functions and more than

twice size of optimal antiderivative.

$$\int \frac{(e \sin [c+d x])^{9/2}}{a+b \cos [c+d x]} d x$$

Optimal (type 4, 461 leaves, 14 steps):

$$\begin{aligned} & -\frac{(-a^2+b^2)^{7/4} e^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{9/2} d} + \frac{(-a^2+b^2)^{7/4} e^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{9/2} d} + \\ & \left(a (a^2-b^2)^2 e^5 \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]} \right) / \\ & \left(b^5 (b-\sqrt{-a^2+b^2}) d \sqrt{e \sin [c+d x]} \right) + \\ & \left(a (a^2-b^2)^2 e^5 \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]} \right) / \\ & \left(b^5 (b+\sqrt{-a^2+b^2}) d \sqrt{e \sin [c+d x]} \right) - \\ & \frac{2 a (5 a^2-8 b^2) e^4 \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{e \sin [c+d x]}}{5 b^4 d \sqrt{\sin [c+d x]}} + \\ & \frac{2 e^3 (5 (a^2-b^2)-3 a b \cos [c+d x]) (e \sin [c+d x])^{3/2}}{15 b^3 d} - \frac{2 e (e \sin [c+d x])^{7/2}}{7 b d} \end{aligned}$$

Result (type 6, 1228 leaves):

$$\begin{aligned} & -\frac{1}{5 b^3 d \sin [c+d x]^{9/2}} (e \sin [c+d x])^{9/2} \\ & \left(\frac{1}{(a+b \cos [c+d x]) (1-\sin [c+d x])^2} 2 (5 a^3-8 a b^2) \cos [c+d x]^2 (a+b \sqrt{1-\sin [c+d x]^2}) \right. \\ & \left. \left(\left(a \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c+d x]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c+d x]}}{(a^2-b^2)^{1/4}}\right] \right) + \operatorname{Log}\left[\right. \right. \right. \\ & \left. \left. \left. \frac{\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin [c+d x]}+b \sin [c+d x]}{\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin [c+d x]}+b \sin [c+d x]} \right] \right) \right) / \\ & \left(4 \sqrt{2} b^{3/2} (a^2-b^2)^{1/4} \right) + \left(7 b (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4},-\frac{1}{2}, 1, \frac{7}{4}, \right. \right. \\ & \left. \left. \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sin [c+d x]^{3/2} \sqrt{1-\sin [c+d x]^2} \right) / \\ & \left(3 \left(-7 (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4},-\frac{1}{2}, 1, \frac{7}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] + 2 \right. \right. \\ & \left. \left. \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4},-\frac{1}{2}, 2, \frac{11}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] + \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
 & \left((a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right. \\
 & \left. \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \Bigg) + \\
 & \frac{1}{12 (a + b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} (2 a^2 b - 5 b^3) \cos[c + dx] \\
 & \left(a + b \sqrt{1 - \sin[c + dx]^2} \right) \\
 & \left(\left((3 + 3i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \right. \right. \right. \\
 & \left. \left. \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} \right. \right. \right. \\
 & \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] + \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1+i) \right. \right. \\
 & \left. \left. \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] \right) \Bigg) / \left(\sqrt{b} (-a^2 + b^2)^{1/4} + \right. \\
 & \left. \left(56 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{3/2} \right) / \right. \\
 & \left(\sqrt{1 - \sin[c + dx]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \right. \right. \\
 & \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \Bigg) \Bigg) + \\
 & \frac{1}{d} \operatorname{Csc}[c + dx]^4 (e \sin[c + dx])^{9/2} \left(-\frac{(-28 a^2 + 37 b^2) \sin[c + dx]}{42 b^3} - \right. \\
 & \left. \frac{a \sin[2(c + dx)]}{5 b^2} + \right. \\
 & \left. \frac{\sin[3(c + dx)]}{14 b} \right)
 \end{aligned}$$

Problem 60: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \sin[c + dx])^{7/2}}{a + b \cos[c + dx]} dx$$

Optimal (type 4, 474 leaves, 14 steps):

$$\frac{(-a^2 + b^2)^{5/4} e^{7/2} \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \text{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{7/2} d} + \frac{(-a^2 + b^2)^{5/4} e^{7/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \text{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{7/2} d} -$$

$$\frac{2 a (3 a^2 - 4 b^2) e^4 \text{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\text{Sin}[c+dx]}}{3 b^4 d \sqrt{e \text{Sin}[c+dx]}} +$$

$$\left(a (a^2 - b^2)^2 e^4 \text{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\text{Sin}[c+dx]} \right) /$$

$$\left(b^4 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2} \right) \right) d \sqrt{e \text{Sin}[c+dx]} \right) +$$

$$\left(a (a^2 - b^2)^2 e^4 \text{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\text{Sin}[c+dx]} \right) /$$

$$\left(b^4 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2} \right) \right) d \sqrt{e \text{Sin}[c+dx]} \right) +$$

$$\frac{2 e^3 (3 (a^2 - b^2) - a b \text{Cos}[c+dx]) \sqrt{e \text{Sin}[c+dx]}}{3 b^3 d} - \frac{2 e (e \text{Sin}[c+dx])^{5/2}}{5 b d}$$

Result (type 6, 2155 leaves):

$$\frac{\left(-\frac{2 a \text{Cos}[c+dx]}{3 b^2} + \frac{\text{Cos}[2(c+dx)]}{5 b} \right) \text{Csc}[c+dx]^3 (e \text{Sin}[c+dx])^{7/2}}{d} +$$

$$\frac{1}{60 b^2 d \text{Sin}[c+dx]^{7/2}} (e \text{Sin}[c+dx])^{7/2}$$

$$\left(\frac{1}{(a + b \text{Cos}[c+dx]) (1 - \text{Sin}[c+dx])^2} 28 a b \text{Cos}[c+dx]^2 \left(a + b \sqrt{1 - \text{Sin}[c+dx]^2} \right) \right.$$

$$\left. \left(\left(a \left(-2 \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c+dx]}}{(a^2 - b^2)^{1/4}}\right] \right) - \right. \right.$$

$$\left. \left. \text{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c+dx]} + b \text{Sin}[c+dx]\right] + \right. \right.$$

$$\left. \left. \text{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c+dx]} + b \text{Sin}[c+dx]\right] \right) \right) /$$

$$\left(4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4} \right) + \left(5 b (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \text{Sin}[c+dx]^2, \right. \right.$$

$$\left. \frac{b^2 \text{Sin}[c+dx]^2}{-a^2 + b^2} \right] \sqrt{\text{Sin}[c+dx]} \sqrt{1 - \text{Sin}[c+dx]^2} \Big) /$$

$$\left(\left(-5 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2 + b^2} \right] + \right. \right.$$

$$\left. 2 \left(2 b^2 \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \right. \right.$$

$$\left. \left. \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2 + b^2} \right] \right) \text{Sin}[c+dx]^2 \right)$$

$$\begin{aligned}
 & \left(10 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c + d x]^2, \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \sqrt{\sin [c + d x]} \right) / \\
 & \left(\sqrt{1 - \sin [c + d x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c + d x]^2, \right. \right. \right. \\
 & \quad \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin [c + d x]^2, \right. \right. \\
 & \quad \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin [c + d x]^2, \right. \\
 & \quad \left. \left. \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \right) \sin [c + d x]^2 \right) (a^2 + b^2 (-1 + \sin [c + d x]^2)) \right) - \\
 & \left(36 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c + d x]^2, \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \sin [c + d x]^{5/2} \right) / \\
 & \left(5 \sqrt{1 - \sin [c + d x]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c + d x]^2, \right. \right. \right. \\
 & \quad \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin [c + d x]^2, \right. \right. \\
 & \quad \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin [c + d x]^2, \right. \\
 & \quad \left. \left. \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \right) \sin [c + d x]^2 \right) (a^2 + b^2 (-1 + \sin [c + d x]^2)) \right) \Big) \Big) \Big)
 \end{aligned}$$

Problem 61: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \sin [c + d x])^{5/2}}{a + b \cos [c + d x]} dx$$

Optimal (type 4, 399 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{(-a^2 + b^2)^{3/4} e^{5/2} \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{5/2} d} + \frac{(-a^2 + b^2)^{3/4} e^{5/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{b^{5/2} d} \\
 & \left(a (a^2 - b^2) e^3 \text{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]} \right) / \\
 & \left(b^3 \left(b - \sqrt{-a^2 + b^2}\right) d \sqrt{e \sin[c+dx]} \right) - \\
 & \left(a (a^2 - b^2) e^3 \text{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]} \right) / \\
 & \left(b^3 \left(b + \sqrt{-a^2 + b^2}\right) d \sqrt{e \sin[c+dx]} \right) + \\
 & \frac{2 a e^2 \text{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{e \sin[c+dx]}}{b^2 d \sqrt{\sin[c+dx]}} - \frac{2 e (e \sin[c+dx])^{3/2}}{3 b d}
 \end{aligned}$$

Result (type 6, 1151 leaves):

$$\begin{aligned}
 & - \frac{2 \text{Csc}[c+dx] (e \sin[c+dx])^{5/2}}{3 b d} + \frac{1}{b d \sin[c+dx]^{5/2}} (e \sin[c+dx])^{5/2} \\
 & \left(\frac{1}{(a + b \cos[c+dx]) (1 - \sin[c+dx])^2} 2 a \cos[c+dx]^2 \left(a + b \sqrt{1 - \sin[c+dx]^2}\right) \right. \\
 & \left. \left(\left(a \left(-2 \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] \right) + \right. \right. \\
 & \left. \left. \text{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] - \right. \right. \\
 & \left. \left. \text{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] \right) \right) / \\
 & \left(4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4} \right) + \left(7 b (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \right. \right. \\
 & \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] \sin[c+dx]^{3/2} \sqrt{1 - \sin[c+dx]^2} \right) / \\
 & \left(3 \left(-7 (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] + \right. \right. \\
 & \left. \left. 2 \left(2 b^2 \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
 & \left. \left. \left. (a^2 - b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] \right) \right) \right) \\
 & \left. \sin[c+dx]^2 \right) (a^2 + b^2 (-1 + \sin[c+dx]^2)) \Big) + \\
 & \frac{1}{12 (a + b \cos[c+dx]) \sqrt{1 - \sin[c+dx]^2}} b \cos[c+dx] \left(a + b \sqrt{1 - \sin[c+dx]^2}\right)
 \end{aligned}$$

$$\left(\left((3 + 3i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i)\sqrt{b}\sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i)\sqrt{b}\sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2+b^2} - (1+i)\sqrt{b} \right. \right. \right. \right. \\ \left. \left. \left. (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + ib \sin[c+dx] \right] + \operatorname{Log} \left[\sqrt{-a^2+b^2} + (1+i) \right. \right. \right. \\ \left. \left. \left. \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + ib \sin[c+dx] \right] \right) \right) / \left(\sqrt{b} (-a^2+b^2)^{1/4} + \right. \\ \left. \left(56 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \sin[c+dx]^{3/2} \right) / \right. \\ \left. \left(\sqrt{1 - \sin[c+dx]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \right. \right. \right. \right. \\ \left. \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \right. \right. \right. \right. \\ \left. \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \right. \right. \right. \\ \left. \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \right) \sin[c+dx]^2 \right) (a^2 + b^2 (-1 + \sin[c+dx]^2)) \right) \right)$$

Problem 62: Result unnecessarily involves higher level functions.

$$\int \frac{(e \sin[c+dx])^{3/2}}{a+b \cos[c+dx]} dx$$

Optimal (type 4, 410 leaves, 13 steps):

$$\frac{(-a^2+b^2)^{1/4} e^{3/2} \operatorname{ArcTan} \left[\frac{\sqrt{b}\sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}} \right]}{b^{3/2} d} + \frac{(-a^2+b^2)^{1/4} e^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b}\sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}} \right]}{b^{3/2} d} + \\ \frac{2 a e^2 \operatorname{EllipticF} \left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{\sin[c+dx]}}{b^2 d \sqrt{e \sin[c+dx]}} - \\ \left(a (a^2 - b^2) e^2 \operatorname{EllipticPi} \left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{\sin[c+dx]} \right) / \\ \left(b^2 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2} \right) \right) d \sqrt{e \sin[c+dx]} \right) - \\ \left(a (a^2 - b^2) e^2 \operatorname{EllipticPi} \left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{\sin[c+dx]} \right) / \\ \left(b^2 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2} \right) \right) d \sqrt{e \sin[c+dx]} \right) - \frac{2 e \sqrt{e \sin[c+dx]}}{b d}$$

Result (type 6, 624 leaves):

$$\begin{aligned}
 & \frac{1}{20 d (a + b \cos [c + d x]) \sin [c + d x]^{3/2} \sqrt{1 - \sin [c + d x]^2}} \\
 & \cos [c + d x] (e \sin [c + d x])^{3/2} \left(a + b \sqrt{1 - \sin [c + d x]^2} \right) \\
 & \left(-\frac{1}{b^{3/2}} (5 - 5 i) \left(2 (-a^2 + b^2)^{1/4} \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\sin [c + d x]}}{(-a^2 + b^2)^{1/4}} \right] - \right. \right. \\
 & \quad \left. \left. 2 (-a^2 + b^2)^{1/4} \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\sin [c + d x]}}{(-a^2 + b^2)^{1/4}} \right] + \right. \right. \\
 & \quad \left. \left. (-a^2 + b^2)^{1/4} \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin [c + d x]} + i b \sin [c + d x] \right] - \right. \right. \\
 & \quad \left. \left. (-a^2 + b^2)^{1/4} \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin [c + d x]} + i b \sin [c + d x] \right] + \right. \right. \\
 & \quad \left. \left. (4 + 4 i) \sqrt{b} \sqrt{\sin [c + d x]} \right) + \right. \\
 & \left. \left(72 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c + d x]^2, \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \sin [c + d x]^{5/2} \right) / \right. \\
 & \left. \left(\sqrt{1 - \sin [c + d x]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c + d x]^2, \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] - \right. \right. \right. \\
 & \quad \left. \left. 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin [c + d x]^2, \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
 & \quad \left. \left. \left. (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin [c + d x]^2, \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \right) \right) \right) \right) \\
 & \left. \left. \sin [c + d x]^2 \right) (a^2 + b^2 (-1 + \sin [c + d x]^2)) \right) \right)
 \end{aligned}$$

Problem 63: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{e \sin [c + d x]}}{a + b \cos [c + d x]} dx$$

Optimal (type 4, 302 leaves, 9 steps):

$$\begin{aligned}
 & -\frac{\sqrt{e} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{e \sin [c + d x]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{\sqrt{b} (-a^2 + b^2)^{1/4} d} + \frac{\sqrt{e} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{e \sin [c + d x]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{\sqrt{b} (-a^2 + b^2)^{1/4} d} + \\
 & \frac{a e \operatorname{EllipticPi} \left[\frac{2b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x \right), 2 \right] \sqrt{\sin [c + d x]}}{b \left(b - \sqrt{-a^2 + b^2} \right) d \sqrt{e \sin [c + d x]}} + \\
 & \frac{a e \operatorname{EllipticPi} \left[\frac{2b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x \right), 2 \right] \sqrt{\sin [c + d x]}}{b \left(b + \sqrt{-a^2 + b^2} \right) d \sqrt{e \sin [c + d x]}}
 \end{aligned}$$

Result (type 6, 556 leaves):

$$\frac{1}{12 d \sqrt{\cos [c+d x]^2 (a+b \cos [c+d x])} \sqrt{e \sin [c+d x]}}$$

$$\cos [c+d x] \left(a+b \sqrt{\cos [c+d x]^2} \right) \sqrt{e \sin [c+d x]}$$

$$\left(\left((3+3 i) \left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}}\right]-2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}}\right]\right)-\right.$$

$$\log \left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b}(-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]+$$

$$\left. \log \left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b}(-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]\right) /$$

$$\left(\sqrt{b}(-a^2+b^2)^{1/4}\right)+\left(56 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]\right.$$

$$\left. \sin [c+d x]^{3/2}\right) / \left(\sqrt{\cos [c+d x]^2\left(a^2-b^2+b^2 \sin [c+d x]^2\right)}\right.$$

$$\left(7\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]-\right.$$

$$2\left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]+$$

$$\left. \left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]\right) \sin [c+d x]^2\left. \right)$$

Problem 64: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+b \cos [c+d x]) \sqrt{e \sin [c+d x]}} d x$$

Optimal (type 4, 307 leaves, 9 steps):

$$\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{3/4} d \sqrt{e}} + \frac{\sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{3/4} d \sqrt{e}} +$$

$$\frac{a \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin [c+d x]}} +$$

$$\frac{a \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin [c+d x]}}$$

Result (type 6, 558 leaves):

$$\begin{aligned}
 & \frac{1}{d \sqrt{\cos [c+d x]^2 (a+b \cos [c+d x])} \sqrt{e \sin [c+d x]}} \\
 & 2 \cos [c+d x] \left(a+b \sqrt{\cos [c+d x]^2} \right) \sqrt{\sin [c+d x]} \left(-\frac{1}{(-a^2+b^2)^{3/4}} \left(\frac{1}{8}-\frac{i}{8} \right) \sqrt{b} \right. \\
 & \quad \left(2 \operatorname{ArcTan} \left[1-\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right]-2 \operatorname{ArcTan} \left[1+\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right] \right) + \\
 & \quad \left(\operatorname{Log} \left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]}+i b \sin [c+d x] \right]-\right. \\
 & \quad \left. \operatorname{Log} \left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]}+i b \sin [c+d x] \right] \right) + \\
 & \quad \left(5 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sqrt{\sin [c+d x]} \right) / \\
 & \quad \left(\sqrt{\cos [c+d x]^2\left(a^2-b^2+b^2 \sin [c+d x]^2\right)}\left(5\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]-2\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \right. \right. \\
 & \quad \left. \left. +\left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \right) \sin [c+d x]^2 \right) \right)
 \end{aligned}$$

Problem 65: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+b \cos [c+d x])\left(e \sin [c+d x]\right)^{3/2}} d x$$

Optimal (type 4, 426 leaves, 13 steps):

$$\begin{aligned}
 & -\frac{b^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{5/4} d e^{3/2}}+\frac{b^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{5/4} d e^{3/2}}+ \\
 & \frac{2(b-a \cos [c+d x])}{\left(a^2-b^2\right) d e \sqrt{e \sin [c+d x]}}-\frac{a b \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{\left(a^2-b^2\right)\left(b-\sqrt{-a^2+b^2}\right) d e \sqrt{e \sin [c+d x]}}- \\
 & \frac{a b \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{\left(a^2-b^2\right)\left(b+\sqrt{-a^2+b^2}\right) d e \sqrt{e \sin [c+d x]}}- \\
 & \frac{2 a \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{e \sin [c+d x]}}{\left(a^2-b^2\right) d e^2 \sqrt{\sin [c+d x]}}
 \end{aligned}$$

Result (type 6, 1186 leaves):

$$\begin{aligned}
 & -\frac{2(-b+a \cos [c+d x]) \sin [c+d x]}{(a^2-b^2) d(e \sin [c+d x])^{3/2}}-\frac{1}{(a-b)(a+b) d(e \sin [c+d x])^{3/2}} \sin [c+d x]^{3/2} \\
 & \left(\frac{1}{(a+b \cos [c+d x])(1-\sin [c+d x]^2)} 2 a b \cos [c+d x]^2\left(a+b \sqrt{1-\sin [c+d x]^2}\right) \right. \\
 & \left. \left(\left(a\left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c+d x]}}{(a^2-b^2)^{1/4}}\right]+2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c+d x]}}{(a^2-b^2)^{1/4}}\right]\right)+\right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1/4} \sqrt{\sin [c+d x]}+b \sin [c+d x]\right]-\right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1/4} \sqrt{\sin [c+d x]}+b \sin [c+d x]\right]\right)\right) / \\
 & \left(4 \sqrt{2} b^{3/2}\left(a^2-b^2\right)^{1/4}\right)+\left(7 b\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4},-\frac{1}{2}, 1, \frac{7}{4}, \sin [c+d x]^2,\right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sin [c+d x]^{3/2} \sqrt{1-\sin [c+d x]^2}\right) / \\
 & \left(3\left(-7\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4},-\frac{1}{2}, 1, \frac{7}{4}, \sin [c+d x]^2,\right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]+2\left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4},-\frac{1}{2}, 2, \frac{11}{4}, \sin [c+d x]^2,\right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right)+\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin [c+d x]^2,\right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]\right) \sin [c+d x]^2\left(a^2+b^2(-1+\sin [c+d x]^2)\right) \left. \right) + \\
 & \frac{1}{12(a+b \cos [c+d x]) \sqrt{1-\sin [c+d x]^2}}\left(a^2+b^2\right) \cos [c+d x] \\
 & \left(a+b \sqrt{1-\sin [c+d x]^2}\right) \\
 & \left(\left((3+3 i)\left(2 \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1/4}}\right]-\right. \right. \right. \\
 & \quad \left. \left. 2 \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1/4}}\right]-\operatorname{Log}\left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b}\right. \right. \right. \\
 & \quad \left. \left. \left(-a^2+b^2\right)^{1/4} \sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]+\operatorname{Log}\left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b}\right. \right. \\
 & \quad \left. \left. \left.\left.\left.\sqrt{b}\left(-a^2+b^2\right)^{1/4} \sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]\right)\right)\right) / \left(\sqrt{b}\left(-a^2+b^2\right)^{1/4}\right)+ \\
 & \left(56 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c+d x]^2,\right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sin [c+d x]^{3/2}\right) / \\
 & \left(\sqrt{1-\sin [c+d x]^2}\left(7\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c+d x]^2,\right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]-2\left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin [c+d x]^2,\right. \right. \right.
 \end{aligned}$$

$$\left. \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + (-a^2+b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^2 \left(a^2+b^2(-1+\sin[c+dx]^2)\right) \right] \right) \right)$$

Problem 66: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+b \cos[c+dx]) (e \sin[c+dx])^{5/2}} dx$$

Optimal (type 4, 447 leaves, 13 steps):

$$\begin{aligned} & \frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{7/4} d e^{5/2}} + \frac{b^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{7/4} d e^{5/2}} + \\ & \frac{2(b-a \cos[c+dx])}{3(a^2-b^2) d e (e \sin[c+dx])^{3/2}} + \frac{2a \operatorname{EllipticF}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right] \sqrt{\sin[c+dx]}}{3(a^2-b^2) d e^2 \sqrt{e \sin[c+dx]}} - \\ & \frac{a b^2 \operatorname{EllipticPi}\left[\frac{-2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right] \sqrt{\sin[c+dx]}}{(a^2-b^2) \left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right) d e^2 \sqrt{e \sin[c+dx]}} - \\ & \frac{a b^2 \operatorname{EllipticPi}\left[\frac{-2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right] \sqrt{\sin[c+dx]}}{(a^2-b^2) \left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right) d e^2 \sqrt{e \sin[c+dx]}} \end{aligned}$$

Result (type 6, 1192 leaves):

$$\begin{aligned} & -\frac{2(-b+a \cos[c+dx]) \sin[c+dx]}{3(a^2-b^2) d (e \sin[c+dx])^{5/2}} + \frac{1}{3(a-b)(a+b) d (e \sin[c+dx])^{5/2}} \sin[c+dx]^{5/2} \\ & \left(\frac{1}{(a+b \cos[c+dx]) (1-\sin[c+dx])^2} 2ab \cos[c+dx]^2 \left(a+b \sqrt{1-\sin[c+dx]^2}\right) \right. \\ & \left. \left(\left(a \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] \right) - \right. \right. \\ & \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]}+b \sin[c+dx]\right] + \right. \right. \\ & \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]}+b \sin[c+dx]\right] \right) \right) / \\ & \left(4 \sqrt{2} \sqrt{b} (a^2-b^2)^{3/4} + \left(5 b (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \sqrt{\sin[c+dx]} \sqrt{1-\sin[c+dx]^2} \right) / \right) \end{aligned}$$

$$\left(\left(-5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \\ \left. \left. 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \right. \right. \right. \\ \left. \left. \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) \right) \\ \left. \left. \left. (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) \right) + \frac{1}{(a + b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} \\ 2 (a^2 - 3 b^2) \cos[c + dx] \left(a + b \sqrt{1 - \sin[c + dx]^2} \right) \left(-\frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \right. \\ \left. \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] \right) + \right. \\ \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] - \right. \\ \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] \right) + \\ \left(5 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c + dx]} \right) / \\ \left(\sqrt{1 - \sin[c + dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \right. \right. \right. \\ \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \right. \right. \right. \\ \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \right. \right. \\ \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) \right)$$

Problem 67: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cos[c + dx]) (e \sin[c + dx])^{7/2}} dx$$

Optimal (type 4, 501 leaves, 14 steps):

$$\begin{aligned}
 & - \frac{b^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{9/4} d e^{7/2}} + \frac{b^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{(-a^2+b^2)^{9/4} d e^{7/2}} + \\
 & \frac{2(b-a \cos[c+dx])}{5(a^2-b^2) d e (e \sin[c+dx])^{5/2}} - \frac{2(5b^3+a(3a^2-8b^2) \cos[c+dx])}{5(a^2-b^2)^2 d e^3 \sqrt{e \sin[c+dx]}} + \\
 & \frac{a b^3 \operatorname{EllipticPi}\left[\frac{2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right] \sqrt{\sin[c+dx]}}{(a^2-b^2)^2 (b-\sqrt{-a^2+b^2}) d e^3 \sqrt{e \sin[c+dx]}} + \\
 & \frac{a b^3 \operatorname{EllipticPi}\left[\frac{2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right] \sqrt{\sin[c+dx]}}{(a^2-b^2)^2 (b+\sqrt{-a^2+b^2}) d e^3 \sqrt{e \sin[c+dx]}} - \\
 & \frac{2a(3a^2-8b^2) \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right), 2\right] \sqrt{e \sin[c+dx]}}{5(a^2-b^2)^2 d e^4 \sqrt{\sin[c+dx]}}
 \end{aligned}$$

Result (type 6, 1275 leaves):

$$\begin{aligned}
 & \left(\left(- \frac{2(5b^3+3a^3 \cos[c+dx]-8ab^2 \cos[c+dx]) \operatorname{Csc}[c+dx]}{5(a^2-b^2)^2} - \right. \right. \\
 & \left. \left. \frac{2(-b+a \cos[c+dx]) \operatorname{Csc}[c+dx]^3}{5(a^2-b^2)} \right) \operatorname{Sin}[c+dx]^4 \right) / \\
 & (d (e \sin[c+dx])^{7/2}) - \frac{1}{5(a-b)^2 (a+b)^2 d (e \sin[c+dx])^{7/2}} \operatorname{Sin}[c+dx]^{7/2} \\
 & \left(\frac{1}{(a+b \cos[c+dx]) (1-\sin[c+dx])^2} 2(3a^3b-8ab^3) \cos[c+dx]^2 (a+b \sqrt{1-\sin[c+dx]^2}) \right. \\
 & \left(\left(a \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] \right) + \right. \\
 & \left. \operatorname{Log}\left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]}+b \sin[c+dx]\right] - \right. \\
 & \left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]}+b \sin[c+dx]\right] \right) \right) / \\
 & (4 \sqrt{2} b^{3/2} (a^2-b^2)^{1/4}) + \left(7b (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \right. \right. \\
 & \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \operatorname{Sin}[c+dx]^{3/2} \sqrt{1-\sin[c+dx]^2} \right) / \\
 & \left(3 \left(-7(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \\
 & \left. \left. 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \right. \\
 & \left. \left. \left. (a^2-b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{9 a (-a^2 + b^2)^{5/4} e^{11/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{11/2} d} + \frac{9 a (-a^2 + b^2)^{5/4} e^{11/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{11/2} d} - \\
 & \left(\frac{3 (21 a^4 - 28 a^2 b^2 + 5 b^4) e^6 \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{7 b^6 d \sqrt{e \sin[c+dx]}} \right) / \\
 & \left(9 a^2 (a^2 - b^2)^2 e^6 \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]} \right) / \\
 & \left(2 b^6 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \sin[c+dx]} \right) + \\
 & \left(9 a^2 (a^2 - b^2)^2 e^6 \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]} \right) / \\
 & \left(2 b^6 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \sin[c+dx]} \right) + \\
 & \frac{3 e^5 (21 a (a^2 - b^2) - b (7 a^2 - 5 b^2) \cos[c+dx]) \sqrt{e \sin[c+dx]}}{7 b^5 d} - \\
 & \frac{9 e^3 (7 a - 5 b \cos[c+dx]) (e \sin[c+dx])^{5/2}}{35 b^3 d} + \frac{e (e \sin[c+dx])^{9/2}}{b d (a + b \cos[c+dx])}
 \end{aligned}$$

Result(type 6, 2229 leaves):

$$\begin{aligned}
 & \frac{1}{d} \left(\frac{(-28 a^2 + 17 b^2) \cos[c+dx]}{14 b^4} + \frac{(-a^2 + b^2)^2}{b^5 (a + b \cos[c+dx])} + \frac{2 a \cos[2(c+dx)]}{5 b^3} - \frac{\cos[3(c+dx)]}{14 b^2} \right) \\
 & \operatorname{Csc}[c+dx]^5 (e \sin[c+dx])^{11/2} - \\
 & \frac{1}{70 b^5 d \sin[c+dx]^{11/2}} (e \sin[c+dx])^{11/2} \left(\frac{1}{(a + b \cos[c+dx]) (1 - \sin[c+dx])^2} \right. \\
 & \left. 2 (35 a^4 - 126 a^2 b^2 + 75 b^4) \cos[c+dx]^2 \left(a + b \sqrt{1 - \sin[c+dx]}\right)^2 \right) \\
 & \left(\left(a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] \right) - \right. \right. \\
 & \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] + \right. \right. \\
 & \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] \right) \right) / \\
 & \left(4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4} + \left(5 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c+dx]} \sqrt{1 - \sin[c+dx]^2} \right) / \\
 & \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] + \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] + (a^2-b^2) \right. \\
 & \quad \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] \right) \sin [c+d x]^2 \Bigg) \\
 & (a^2+b^2 (-1+\sin [c+d x]^2)) \Bigg) + \frac{1}{(a+b \cos [c+d x]) \sqrt{1-\sin [c+d x]^2}} \\
 & 2 (70 a^3 b - 93 a b^3) \cos [c+d x] \left(a+b \sqrt{1-\sin [c+d x]^2} \right) \left(-\frac{1}{(-a^2+b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \right. \\
 & \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right] \right) + \\
 & \left. \operatorname{Log} \left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]} + i b \sin [c+d x] \right] - \right. \\
 & \left. \operatorname{Log} \left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]} + i b \sin [c+d x] \right] \right) + \\
 & \left(5 a (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] \sqrt{\sin [c+d x]} \right) / \\
 & \left(\sqrt{1-\sin [c+d x]^2} \left(5 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] \right) \sin [c+d x]^2 \Bigg) (a^2+b^2 (-1+\sin [c+d x]^2)) \Bigg) \Bigg) + \\
 & \frac{1}{(a+b \cos [c+d x]) (1-2 \sin [c+d x]^2) \sqrt{1-\sin [c+d x]^2}} (-140 a^3 b + 147 a b^3) \\
 & \cos [c+d x] \cos [2(c+d x)] \\
 & \left(a+b \sqrt{1-\sin [c+d x]^2} \right) \\
 & \left(\frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2+b^2) \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right]}{b^{3/2} (-a^2+b^2)^{3/4}} \right) - \\
 & \frac{\left(\frac{1}{2} - \frac{i}{2} \right) (-2 a^2+b^2) \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right]}{b^{3/2} (-a^2+b^2)^{3/4}} + \left(\left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2+b^2) \right. \\
 & \left. \operatorname{Log} \left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]} + i b \sin [c+d x] \right] \right) / \\
 & \left(b^{3/2} (-a^2+b^2)^{3/4} \right) - \left(\left(\frac{1}{4} - \frac{i}{4} \right) (-2 a^2+b^2) \operatorname{Log} \left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \right] \right)
 \end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{\sin[c+dx]} + i b \sin[c+dx] \right) / \left(b^{3/2} (-a^2 + b^2)^{3/4} + \frac{4 \sqrt{\sin[c+dx]}}{b} + \right. \\
& \left(10 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c+dx]} \right) / \\
& \left(\sqrt{1 - \sin[c+dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \right. \right. \right. \\
& \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \right. \right. \right. \\
& \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \right. \right. \\
& \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] \right) \sin[c+dx]^2 \left(a^2 + b^2 (-1 + \sin[c+dx]^2) \right) \Bigg) - \\
& \left(36 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] \sin[c+dx]^{5/2} \right) / \\
& \left(5 \sqrt{1 - \sin[c+dx]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \right. \right. \right. \\
& \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin[c+dx]^2, \right. \right. \right. \\
& \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin[c+dx]^2, \right. \right. \\
& \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] \right) \sin[c+dx]^2 \left(a^2 + b^2 (-1 + \sin[c+dx]^2) \right) \Bigg) \Bigg)
\end{aligned}$$

Problem 69: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \sin[c+dx])^{9/2}}{(a+b \cos[c+dx])^2} dx$$

Optimal (type 4, 473 leaves, 14 steps):

$$\begin{aligned}
 & - \frac{7 a (-a^2 + b^2)^{3/4} e^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{9/2} d} + \frac{7 a (-a^2 + b^2)^{3/4} e^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{9/2} d} \\
 & \left(\frac{7 a^2 (a^2 - b^2) e^5 \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{2 b^5 \left(b - \sqrt{-a^2 + b^2}\right) d \sqrt{e \operatorname{Sin}[c+dx]}} - \right. \\
 & \left. \frac{7 a^2 (a^2 - b^2) e^5 \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\operatorname{Sin}[c+dx]}}{2 b^5 \left(b + \sqrt{-a^2 + b^2}\right) d \sqrt{e \operatorname{Sin}[c+dx]}} \right) + \\
 & \frac{7 (5 a^2 - 3 b^2) e^4 \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{e \operatorname{Sin}[c+dx]}}{5 b^4 d \sqrt{\operatorname{Sin}[c+dx]}} - \\
 & \frac{7 e^3 (5 a - 3 b \operatorname{Cos}[c+dx]) (e \operatorname{Sin}[c+dx])^{3/2}}{15 b^3 d} + \frac{e (e \operatorname{Sin}[c+dx])^{7/2}}{b d (a + b \operatorname{Cos}[c+dx])}
 \end{aligned}$$

Result (type 6, 1229 leaves):

$$\begin{aligned}
 & \frac{1}{10 b^3 d \operatorname{Sin}[c+dx]^{9/2}} 7 (e \operatorname{Sin}[c+dx])^{9/2} \\
 & \left(\frac{1}{(a + b \operatorname{Cos}[c+dx]) (1 - \operatorname{Sin}[c+dx]^2)} 2 (5 a^2 - 3 b^2) \operatorname{Cos}[c+dx]^2 \left(a + b \sqrt{1 - \operatorname{Sin}[c+dx]^2} \right) \right. \\
 & \left(\left(a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+dx]}}{(a^2 - b^2)^{1/4}}\right] \right) + \right. \\
 & \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + b \operatorname{Sin}[c+dx]\right] - \right. \right. \\
 & \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c+dx]} + b \operatorname{Sin}[c+dx]\right] \right) \right) / \\
 & \left(4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4} \right) + \left(7 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \right. \right. \\
 & \left. \left. \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2 + b^2} \right] \operatorname{Sin}[c+dx]^{3/2} \sqrt{1 - \operatorname{Sin}[c+dx]^2} \right) / \\
 & \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2 + b^2} \right] + \right. \right. \\
 & \left. \left. 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
 & \left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c+dx]^2, \frac{b^2 \operatorname{Sin}[c+dx]^2}{-a^2 + b^2} \right] \right) \right) \\
 & \left. \operatorname{Sin}[c+dx]^2 \right) (a^2 + b^2 (-1 + \operatorname{Sin}[c+dx]^2)) \left. \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{6 (a + b \cos [c + d x]) \sqrt{1 - \sin [c + d x]^2}} a b \cos [c + d x] \left(a + b \sqrt{1 - \sin [c + d x]^2} \right) \\
 & \left(\left((3 + 3 i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\sin [c + d x]}}{(-a^2 + b^2)^{1/4}} \right] - \right. \right. \right. \\
 & \quad \left. \left. 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\sin [c + d x]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} \right. \right. \right. \\
 & \quad \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin [c + d x]} + i b \sin [c + d x] \right] + \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \right. \right. \\
 & \quad \left. \left. \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin [c + d x]} + i b \sin [c + d x] \right] \right) \left) \right) / \left(\sqrt{b} (-a^2 + b^2)^{1/4} \right) + \\
 & \left(56 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c + d x]^2, \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \sin [c + d x]^{3/2} \right) / \\
 & \left(\sqrt{1 - \sin [c + d x]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c + d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin [c + d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin [c + d x]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \right) \sin [c + d x]^2 \left(a^2 + b^2 (-1 + \sin [c + d x]^2) \right) \right) \right) \right) + \frac{1}{d} \\
 & \operatorname{Csc} [c + d x]^4 (e \sin [c + d x])^{9/2} \left(-\frac{4 a \sin [c + d x]}{3 b^3} + \frac{-a^2 \sin [c + d x] + b^2 \sin [c + d x]}{b^3 (a + b \cos [c + d x])} + \right. \\
 & \quad \left. \frac{\sin [2 (c + d x)]}{5 b^2} \right)
 \end{aligned}$$

Problem 70: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \sin [c + d x])^{7/2}}{(a + b \cos [c + d x])^2} dx$$

Optimal (type 4, 487 leaves, 14 steps):

$$\frac{5 a (-a^2 + b^2)^{1/4} e^{7/2} \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \text{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{7/2} d} + \frac{5 a (-a^2 + b^2)^{1/4} e^{7/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \text{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{7/2} d} +$$

$$\frac{5 (3 a^2 - b^2) e^4 \text{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\text{Sin}[c+dx]}}{3 b^4 d \sqrt{e \text{Sin}[c+dx]}} -$$

$$\left(5 a^2 (a^2 - b^2) e^4 \text{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\text{Sin}[c+dx]}\right) /$$

$$\left(2 b^4 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \text{Sin}[c+dx]}\right) -$$

$$\left(5 a^2 (a^2 - b^2) e^4 \text{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\text{Sin}[c+dx]}\right) /$$

$$\left(2 b^4 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \text{Sin}[c+dx]}\right) -$$

$$\frac{5 e^3 (3 a - b \text{Cos}[c+dx]) \sqrt{e \text{Sin}[c+dx]}}{3 b^3 d} + \frac{e (e \text{Sin}[c+dx])^{5/2}}{b d (a + b \text{Cos}[c+dx])}$$

Result (type 6, 2156 leaves):

$$\frac{\left(\frac{2 \text{Cos}[c+dx]}{3 b^2} + \frac{-a^2+b^2}{b^3 (a+b \text{Cos}[c+dx])}\right) \text{Csc}[c+dx]^3 (e \text{Sin}[c+dx])^{7/2}}{d} +$$

$$\frac{1}{6 b^3 d \text{Sin}[c+dx]^{7/2}} (e \text{Sin}[c+dx])^{7/2}$$

$$\left(\frac{1}{(a + b \text{Cos}[c+dx]) (1 - \text{Sin}[c+dx])^2} 2 (3 a^2 - 5 b^2) \text{Cos}[c+dx]^2 \left(a + b \sqrt{1 - \text{Sin}[c+dx]^2}\right)\right.$$

$$\left.\left(\left(a \left(-2 \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c+dx]}}{(a^2 - b^2)^{1/4}}\right]\right) - \right.$$

$$\left.\text{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c+dx]} + b \text{Sin}[c+dx]\right] + \right.$$

$$\left.\text{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c+dx]} + b \text{Sin}[c+dx]\right]\right) /$$

$$\left(4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}\right) + \left(5 b (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \text{Sin}[c+dx]^2,\right.\right.$$

$$\left.\frac{b^2 \text{Sin}[c+dx]^2}{-a^2 + b^2}\right] \sqrt{\text{Sin}[c+dx]} \sqrt{1 - \text{Sin}[c+dx]^2}\right) /$$

$$\left(\left(-5 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2 + b^2}\right] + \right.\right.$$

$$\left.2 \left(2 b^2 \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2 + b^2}\right] + (a^2 - b^2) \right.\right.$$

$$\left.\left.\text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2 + b^2}\right]\right) \text{Sin}[c+dx]^2\right)$$

$$\left(10 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \sqrt{\sin[c + dx]} \right) /$$

$$\left(\sqrt{1 - \sin[c + dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) -$$

$$\left(36 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \sin[c + dx]^{5/2} \right) /$$

$$\left(5 \sqrt{1 - \sin[c + dx]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) \right)$$

Problem 71: Result unnecessarily involves higher level functions.

$$\int \frac{(e \sin[c + dx])^{5/2}}{(a + b \cos[c + dx])^2} dx$$

Optimal (type 4, 404 leaves, 13 steps):

$$-\frac{3 a e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{2 b^{5/2} (-a^2 + b^2)^{1/4} d} + \frac{3 a e^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{2 b^{5/2} (-a^2 + b^2)^{1/4} d} +$$

$$\frac{3 a^2 e^3 \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{2 b^3 \left(b - \sqrt{-a^2 + b^2}\right) d \sqrt{e \sin[c + dx]}} +$$

$$-\frac{3 a^2 e^3 \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{2 b^3 \left(b + \sqrt{-a^2 + b^2}\right) d \sqrt{e \sin[c + dx]}} -$$

$$\frac{3 e^2 \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{e \sin[c + dx]}}{b^2 d \sqrt{\sin[c + dx]}} + \frac{e (e \sin[c + dx])^{3/2}}{b d (a + b \cos[c + dx])}$$

Result (type 6, 616 leaves):

$$\frac{\text{Csc}[c + d x] (e \text{Sin}[c + d x])^{5/2}}{b d (a + b \text{Cos}[c + d x])} - \frac{1}{b d (a + b \text{Cos}[c + d x]) \text{Sin}[c + d x]^{5/2} (1 - \text{Sin}[c + d x]^2)}$$

$$3 \text{Cos}[c + d x]^2 (e \text{Sin}[c + d x])^{5/2} (a + b \sqrt{1 - \text{Sin}[c + d x]^2})$$

$$\left(\left(a \left(-2 \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + 2 \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + \right. \right.$$

$$\left. \left. \text{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c + d x]} + b \text{Sin}[c + d x]\right] - \right. \right.$$

$$\left. \left. \text{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c + d x]} + b \text{Sin}[c + d x]\right] \right) \right) /$$

$$\left(4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4} + \left(7 b (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c + d x]^2, \right. \right. \right.$$

$$\left. \left. \frac{b^2 \text{Sin}[c + d x]^2}{-a^2 + b^2} \right] \text{Sin}[c + d x]^{3/2} \sqrt{1 - \text{Sin}[c + d x]^2} \right) /$$

$$\left(3 \left(-7 (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c + d x]^2, \frac{b^2 \text{Sin}[c + d x]^2}{-a^2 + b^2} \right] + \right. \right.$$

$$2 \left(2 b^2 \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \text{Sin}[c + d x]^2, \frac{b^2 \text{Sin}[c + d x]^2}{-a^2 + b^2} \right] + \right.$$

$$\left. \left. (a^2 - b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \text{Sin}[c + d x]^2, \frac{b^2 \text{Sin}[c + d x]^2}{-a^2 + b^2} \right] \right) \right)$$

$$\text{Sin}[c + d x]^2 (a^2 + b^2 (-1 + \text{Sin}[c + d x]^2))) \right)$$

Problem 72: Result unnecessarily involves higher level functions.

$$\int \frac{(e \text{Sin}[c + d x])^{3/2}}{(a + b \text{Cos}[c + d x])^2} dx$$

Optimal (type 4, 418 leaves, 13 steps):

$$\frac{a e^{3/2} \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{3/2} (-a^2+b^2)^{3/4} d} + \frac{a e^{3/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 b^{3/2} (-a^2+b^2)^{3/4} d} -$$

$$\frac{e^2 \text{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{b^2 d \sqrt{e \sin[c+dx]}} +$$

$$\frac{a^2 e^2 \text{EllipticPi}\left[\frac{-2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{2 b^2 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin[c+dx]}} +$$

$$\frac{a^2 e^2 \text{EllipticPi}\left[\frac{-2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{2 b^2 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin[c+dx]}} + \frac{e \sqrt{e \sin[c+dx]}}{b d (a + b \cos[c+dx])}$$

Result(type 6, 614 leaves):

$$\frac{\text{Csc}[c+dx] (e \sin[c+dx])^{3/2}}{b d (a + b \cos[c+dx])} - \frac{1}{b d (a + b \cos[c+dx]) \sin[c+dx]^{3/2} (1 - \sin[c+dx]^2)}$$

$$\text{Cos}[c+dx]^2 (e \sin[c+dx])^{3/2} \left(a + b \sqrt{1 - \sin[c+dx]^2}\right)$$

$$\left(\left(a \left(-2 \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right]\right) - \right.$$

$$\left. \text{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] + \right.$$

$$\left. \text{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right]\right) \Bigg/$$

$$\left(4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4}\right) + \left(5 b (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2}\right], \right.$$

$$\left. \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \sqrt{\sin[c+dx]} \sqrt{1 - \sin[c+dx]^2}\right) \Bigg/$$

$$\left(\left(-5 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2}\right] + \right.\right.$$

$$\left. 2 \left(2 b^2 \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2}\right] + \right.\right.$$

$$\left. \left.(a^2 - b^2) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2}\right]\right) \right)$$

$$\sin[c+dx]^2 (a^2 + b^2 (-1 + \sin[c+dx]^2)) \Bigg)$$

Problem 73: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{e \sin[c+dx]}}{(a+b \cos[c+dx])^2} dx$$

Optimal (type 4, 438 leaves, 13 steps):

$$\begin{aligned} & \frac{a \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 \sqrt{b} (-a^2+b^2)^{5/4} d} - \frac{a \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2 \sqrt{b} (-a^2+b^2)^{5/4} d} + \\ & \frac{a^2 e \operatorname{EllipticPi}\left[\frac{-2b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{2b(a^2-b^2)(b-\sqrt{-a^2+b^2})d\sqrt{e \sin[c+dx]}} + \\ & \frac{a^2 e \operatorname{EllipticPi}\left[\frac{-2b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{2b(a^2-b^2)(b+\sqrt{-a^2+b^2})d\sqrt{e \sin[c+dx]}} + \\ & \frac{\operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{e \sin[c+dx]}}{(a^2-b^2)d\sqrt{\sin[c+dx]}} - \frac{b(e \sin[c+dx])^{3/2}}{(a^2-b^2)de(a+b \cos[c+dx])} \end{aligned}$$

Result (type 6, 1181 leaves):

$$\begin{aligned} & \frac{b \sin[c+dx] \sqrt{e \sin[c+dx]}}{(-a^2+b^2)d(a+b \cos[c+dx])} + \frac{1}{2(a-b)(a+b)d\sqrt{\sin[c+dx]}} \sqrt{e \sin[c+dx]} \\ & \left(\frac{1}{(a+b \cos[c+dx])(1-\sin[c+dx])^2} 2b \cos[c+dx]^2 (a+b \sqrt{1-\sin[c+dx]^2}) \right. \\ & \left. \left(\left(a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}}\right] \right) + \right. \right. \\ & \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] - \right. \right. \\ & \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] \right) \right) / \\ & \left(4 \sqrt{2} b^{3/2} (a^2-b^2)^{1/4} + \left(7b(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2\right], \right. \right. \\ & \quad \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right) \sin[c+dx]^{3/2} \sqrt{1-\sin[c+dx]^2} \right) / \\ & \left(3 \left(-7(a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \\ & \quad \left. \left. 2 \left(2b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2}\right] + \right. \right. \end{aligned}$$

$$\begin{aligned}
& \left((a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \right. \\
& \left. \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \Bigg) + \\
& \frac{1}{6 (a + b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} a \cos[c + dx] \left(a + b \sqrt{1 - \sin[c + dx]^2} \right) \\
& \left(\left((3 + 3i) \left(2 \operatorname{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] - \right. \right. \right. \\
& \left. \left. \left. 2 \operatorname{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] - \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} \right. \right. \right. \right. \\
& \left. \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] + \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1+i) \right. \right. \right. \\
& \left. \left. \left. \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] \right) \right) / \left(\sqrt{b} (-a^2 + b^2)^{1/4} \right) + \\
& \left(56 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \sin[c + dx]^{3/2} \right) / \\
& \left(\sqrt{1 - \sin[c + dx]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \right. \right. \right. \\
& \left. \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \Bigg) \Bigg)
\end{aligned}$$

Problem 74: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cos[c + dx])^2 \sqrt{e \sin[c + dx]}} dx$$

Optimal (type 4, 445 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{3 a \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4} \sqrt{e}}\right]}{2\left(-a^2+b^2\right)^{7 / 4} d \sqrt{e}} \\
 & - \frac{3 a \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4} \sqrt{e}}\right]}{2\left(-a^2+b^2\right)^{7 / 4} d \sqrt{e}} - \frac{\operatorname{EllipticF}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{\left(a^2-b^2\right) d \sqrt{e \sin [c+d x]}} + \\
 & \frac{3 a^2 \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin [c+d x]}} + \\
 & \frac{3 a^2 \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{2\left(a^2-b^2\right)\left(a^2-b\left(b+\sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin [c+d x]}} - \frac{b \sqrt{e \sin [c+d x]}}{\left(a^2-b^2\right) d e\left(a+b \cos [c+d x]\right)}
 \end{aligned}$$

Result (type 6, 1182 leaves):

$$\begin{aligned}
 & - \frac{b \sin [c+d x]}{\left(a^2-b^2\right) d\left(a+b \cos [c+d x]\right) \sqrt{e \sin [c+d x]}} + \frac{1}{2(a-b)(a+b) d \sqrt{e \sin [c+d x]}} \sqrt{\sin [c+d x]} \\
 & \left(- \frac{1}{(a+b \cos [c+d x])\left(1-\sin [c+d x]\right)^2} 2 b \cos [c+d x]^2\left(a+b \sqrt{1-\sin [c+d x]^2}\right) \right. \\
 & \left. \left(\left(a \left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c+d x]}}{\left(a^2-b^2\right)^{1 / 4}}\right]+2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c+d x]}}{\left(a^2-b^2\right)^{1 / 4}}\right]\right) - \operatorname{Log}\left[\frac{\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1 / 4} \sqrt{\sin [c+d x]}+b \sin [c+d x]}{\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{1 / 4} \sqrt{\sin [c+d x]}+b \sin [c+d x]}\right] \right) \right) / \\
 & \left(4 \sqrt{2} \sqrt{b}\left(a^2-b^2\right)^{3 / 4}+\left(5 b\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4},-\frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sqrt{\sin [c+d x]} \sqrt{1-\sin [c+d x]^2}\right) \right) / \\
 & \left(\left(-5\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4},-\frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]+2\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4},-\frac{1}{2}, 2, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]+\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]\right) \right) \right) \sqrt{\sin [c+d x]^2}\left(a^2+b^2\left(-1+\sin [c+d x]^2\right)\right) \right) \left. \right) + \\
 & \frac{1}{(a+b \cos [c+d x]) \sqrt{1-\sin [c+d x]^2}} 4 a \cos [c+d x]\left(a+b \sqrt{1-\sin [c+d x]^2}\right)
 \end{aligned}$$

$$\left(-\frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \right. \\ \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2 + b^2)^{1/4}} \right] \right) + \\ \left(\operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx] \right] - \right. \\ \left. \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1+i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx] \right] \right) + \\ \left(5 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c+dx]} \right) / \\ \left(\sqrt{1 - \sin[c+dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \right. \right. \right. \\ \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \right. \right. \right. \\ \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \right. \right. \\ \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2 + b^2} \right] \right) \sin[c+dx]^2 (a^2 + b^2 (-1 + \sin[c+dx]^2)) \right) \Bigg)$$

Problem 75: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cos[c + dx])^2 (e \sin[c + dx])^{3/2}} dx$$

Optimal (type 4, 507 leaves, 14 steps):

$$\frac{5 a b^{3/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2 (-a^2 + b^2)^{9/4} d e^{3/2}} - \frac{5 a b^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{2 (-a^2 + b^2)^{9/4} d e^{3/2}} - \\ \frac{b}{(a^2 - b^2) d e (a + b \cos[c + dx]) \sqrt{e \sin[c + dx]}} + \frac{5 a b - (2 a^2 + 3 b^2) \cos[c + dx]}{(a^2 - b^2)^2 d e \sqrt{e \sin[c + dx]}} - \\ \frac{5 a^2 b \operatorname{EllipticPi} \left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{\sin[c + dx]}}{2 (a^2 - b^2)^2 (b - \sqrt{-a^2 + b^2}) d e \sqrt{e \sin[c + dx]}} - \\ \frac{5 a^2 b \operatorname{EllipticPi} \left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{\sin[c + dx]}}{2 (a^2 - b^2)^2 (b + \sqrt{-a^2 + b^2}) d e \sqrt{e \sin[c + dx]}} - \\ \frac{(2 a^2 + 3 b^2) \operatorname{EllipticE} \left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx \right), 2 \right] \sqrt{e \sin[c + dx]}}{(a^2 - b^2)^2 d e^2 \sqrt{\sin[c + dx]}}$$

Result (type 6, 1259 leaves):

$$\begin{aligned}
 & \left(\text{Sin}[c + d x]^2 \right. \\
 & \left. \left(-\frac{2(-2ab + a^2 \text{Cos}[c + dx] + b^2 \text{Cos}[c + dx]) \text{Csc}[c + dx]}{(a^2 - b^2)^2} + \frac{b^3 \text{Sin}[c + dx]}{(a^2 - b^2)^2 (a + b \text{Cos}[c + dx])} \right) \right) / \\
 & \left(d (e \text{Sin}[c + dx])^{3/2} \right) - \frac{1}{2(a-b)^2 (a+b)^2 d (e \text{Sin}[c + dx])^{3/2}} \text{Sin}[c + dx]^{3/2} \\
 & \left(\frac{1}{(a + b \text{Cos}[c + dx]) (1 - \text{Sin}[c + dx]^2)} 2(2a^2 b + 3b^3) \text{Cos}[c + dx]^2 (a + b \sqrt{1 - \text{Sin}[c + dx]^2}) \right. \\
 & \left. \left(\left(a \left(-2 \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c + dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c + dx]}}{(a^2 - b^2)^{1/4}}\right] \right) + \right. \right. \\
 & \quad \text{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c + dx]} + b \text{Sin}[c + dx]\right] - \\
 & \quad \left. \left. \text{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c + dx]} + b \text{Sin}[c + dx]\right] \right) \right) / \\
 & \left(4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4} \right) + \left(7 b (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c + dx]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2} \right] \text{Sin}[c + dx]^{3/2} \sqrt{1 - \text{Sin}[c + dx]^2} \right) / \\
 & \left(3 \left(-7 (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \\
 & \quad 2 \left(2 b^2 \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2} \right] + \right. \\
 & \quad \left. \left. (a^2 - b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2} \right] \right) \right) \\
 & \quad \left. \text{Sin}[c + dx]^2 \right) (a^2 + b^2 (-1 + \text{Sin}[c + dx]^2)) \left. \right) + \\
 & \frac{1}{12(a + b \text{Cos}[c + dx]) \sqrt{1 - \text{Sin}[c + dx]^2}} (2a^3 + 8ab^2) \text{Cos}[c + dx] \\
 & \left(a + b \sqrt{1 - \text{Sin}[c + dx]^2} \right) \\
 & \left(\left((3 + 3i) \left(2 \text{ArcTan}\left[1 - \frac{(1+i) \sqrt{b} \sqrt{\text{Sin}[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] - \right. \right. \right. \\
 & \quad \left. \left. 2 \text{ArcTan}\left[1 + \frac{(1+i) \sqrt{b} \sqrt{\text{Sin}[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] - \text{Log}\left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} \right. \right. \right. \\
 & \quad \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\text{Sin}[c + dx]} + i b \text{Sin}[c + dx]\right] + \text{Log}\left[\sqrt{-a^2 + b^2} + (1+i) \right. \right. \\
 & \quad \left. \left. \left. \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\text{Sin}[c + dx]} + i b \text{Sin}[c + dx]\right] \right) \right) / \left(\sqrt{b} (-a^2 + b^2)^{1/4} \right) +
 \end{aligned}$$

$$\left(56 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \sin[c + dx]^{3/2} \right) / \left(\sqrt{1 - \sin[c + dx]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2}\right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right)$$

Problem 76: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cos[c + dx])^2 (e \sin[c + dx])^{5/2}} dx$$

Optimal (type 4, 530 leaves, 14 steps):

$$\begin{aligned} & - \frac{7 a b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{2 (-a^2 + b^2)^{11/4} d e^{5/2}} - \frac{7 a b^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c + dx]}}{(-a^2 + b^2)^{1/4} \sqrt{e}}\right]}{2 (-a^2 + b^2)^{11/4} d e^{5/2}} \\ & + \frac{b}{(a^2 - b^2) d e (a + b \cos[c + dx]) (e \sin[c + dx])^{3/2}} + \frac{7 a b - (2 a^2 + 5 b^2) \cos[c + dx]}{3 (a^2 - b^2)^2 d e (e \sin[c + dx])^{3/2}} \\ & - \frac{(2 a^2 + 5 b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{3 (a^2 - b^2)^2 d e^2 \sqrt{e \sin[c + dx]}} \\ & - \frac{7 a^2 b^2 \operatorname{EllipticPi}\left[\frac{-2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{2 (a^2 - b^2)^2 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) d e^2 \sqrt{e \sin[c + dx]}} \\ & - \frac{7 a^2 b^2 \operatorname{EllipticPi}\left[\frac{-2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}}{2 (a^2 - b^2)^2 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) d e^2 \sqrt{e \sin[c + dx]}} \end{aligned}$$

Result (type 6, 1257 leaves):

$$\left(\left(\frac{b^3}{(a^2 - b^2)^2 (a + b \cos[c + dx])} - \frac{2 (-2 a b + a^2 \cos[c + dx] + b^2 \cos[c + dx]) \operatorname{Csc}[c + dx]^2}{3 (a^2 - b^2)^2} \right) \sin[c + dx]^3 \right) / \left(d (e \sin[c + dx])^{5/2} \right) + \frac{1}{6 (a - b)^2 (a + b)^2 d (e \sin[c + dx])^{5/2}} \sin[c + dx]^{5/2}$$

$$\begin{aligned}
 & \left(\frac{1}{(a+b \cos [c+d x]) (1-\sin [c+d x]^2)} 2 (2 a^2 b+5 b^3) \cos [c+d x]^2 \left(a+b \sqrt{1-\sin [c+d x]^2} \right) \right. \\
 & \left. \left(\left(a \left(-2 \operatorname{ArcTan} \left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c+d x]}}{(a^2-b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c+d x]}}{(a^2-b^2)^{1/4}} \right] - \right. \right. \right. \\
 & \quad \left. \left. \left. \begin{aligned} & \operatorname{Log} \left[\sqrt{a^2-b^2}-\sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin [c+d x]}+b \sin [c+d x] \right] + \right. \right. \\ & \left. \left. \left. \operatorname{Log} \left[\sqrt{a^2-b^2}+\sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin [c+d x]}+b \sin [c+d x] \right] \right) \right) \right) / \right. \\
 & \quad \left(4 \sqrt{2} \sqrt{b} (a^2-b^2)^{3/4} \right) + \left(5 b (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] \sqrt{\sin [c+d x]} \sqrt{1-\sin [c+d x]^2} \right) / \\
 & \quad \left(\left(-5 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] + \right. \right. \\
 & \quad \left. \left. 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] + (a^2-b^2) \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] \right) \sin [c+d x]^2 \right) \right) \\
 & \quad \left. \left. \left. \left(a^2+b^2 (-1+\sin [c+d x]^2) \right) \right) \right) \right) + \frac{1}{(a+b \cos [c+d x]) \sqrt{1-\sin [c+d x]^2}} \\
 & 2 (2 a^3-16 a b^2) \cos [c+d x] \left(a+b \sqrt{1-\sin [c+d x]^2} \right) \left(-\frac{1}{(-a^2+b^2)^{3/4}} \left(\frac{1}{8}-\frac{i}{8} \right) \sqrt{b} \right. \\
 & \quad \left(2 \operatorname{ArcTan} \left[1-\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1+\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right] + \right. \\
 & \quad \left. \operatorname{Log} \left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]}+i b \sin [c+d x] \right] - \right. \\
 & \quad \left. \left. \operatorname{Log} \left[\sqrt{-a^2+b^2}+(1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]}+i b \sin [c+d x] \right] \right) \right) + \\
 & \quad \left(5 a (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] \sqrt{\sin [c+d x]} \right) / \\
 & \quad \left(\sqrt{1-\sin [c+d x]^2} \left(5 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] \right) \sin [c+d x]^2 \right) \left(a^2+b^2 (-1+\sin [c+d x]^2) \right) \right) \right) \right)
 \end{aligned}$$

Problem 77: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cos [c + d x])^2 (e \sin [c + d x])^{7/2}} dx$$

Optimal (type 4, 590 leaves, 15 steps):

$$\frac{9 a b^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2\left(-a^2+b^2\right)^{13/4} d e^{7/2}} - \frac{9 a b^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{2\left(-a^2+b^2\right)^{13/4} d e^{7/2}} -$$

$$\frac{b}{\left(a^2-b^2\right) d e\left(a+b \cos [c+d x]\right)\left(e \sin [c+d x]\right)^{5/2}} +$$

$$\frac{9 a b - \left(2 a^2+7 b^2\right) \cos [c+d x]}{5\left(a^2-b^2\right)^2 d e\left(e \sin [c+d x]\right)^{5/2}} - \frac{3\left(15 a b^3+\left(2 a^4-10 a^2 b^2-7 b^4\right) \cos [c+d x]\right)}{5\left(a^2-b^2\right)^3 d e^3 \sqrt{e \sin [c+d x]}} +$$

$$\frac{9 a^2 b^3 \operatorname{EllipticPi}\left[\frac{-2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{2\left(a^2-b^2\right)^3\left(b-\sqrt{-a^2+b^2}\right) d e^3 \sqrt{e \sin [c+d x]}} +$$

$$\frac{9 a^2 b^3 \operatorname{EllipticPi}\left[\frac{-2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\sin [c+d x]}}{2\left(a^2-b^2\right)^3\left(b+\sqrt{-a^2+b^2}\right) d e^3 \sqrt{e \sin [c+d x]}} -$$

$$\left(\frac{3\left(2 a^4-10 a^2 b^2-7 b^4\right) \operatorname{EllipticE}\left[\frac{1}{2}\left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{e \sin [c+d x]}}{5\left(a^2-b^2\right)^3 d e^4 \sqrt{\sin [c+d x]}}\right) /$$

Result (type 6, 1344 leaves):

$$\left(\sin [c+d x]^4\left(-\frac{1}{5\left(a^2-b^2\right)^3}\right.\right.$$

$$\left.\left.\frac{2\left(20 a b^3+3 a^4 \cos [c+d x]-15 a^2 b^2 \cos [c+d x]-8 b^4 \cos [c+d x]\right) \operatorname{Csc}[c+d x]-2\left(-2 a b+a^2 \cos [c+d x]+b^2 \cos [c+d x]\right) \operatorname{Csc}[c+d x]^3}{5\left(a^2-b^2\right)^2}-\frac{b^5 \sin [c+d x]}{\left(a^2-b^2\right)^3\left(a+b \cos [c+d x]\right)}\right)\right) /$$

$$\left(d\left(e \sin [c+d x]\right)^{7/2}\right)-\frac{1}{10\left(a-b\right)^3\left(a+b\right)^3 d\left(e \sin [c+d x]\right)^{7/2}}$$

$$3 \sin [c+d x]^{7/2}\left(\frac{1}{\left(a+b \cos [c+d x]\right)\left(1-\sin [c+d x]^2\right)}\right.$$

$$\left.\frac{2\left(2 a^4 b-10 a^2 b^3-7 b^5\right) \cos [c+d x]^2\left(a+b \sqrt{1-\sin [c+d x]^2}\right)}{\left(\left(a\left(-2 \operatorname{ArcTan}\left[1-\frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c+d x]}}{\left(a^2-b^2\right)^{1/4}}\right]+2 \operatorname{ArcTan}\left[1+\frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c+d x]}}{\left(a^2-b^2\right)^{1/4}}\right]\right)+\right.\right.$$

$$\begin{aligned}
 & \left. \left(\frac{\text{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c + dx]} + b \text{Sin}[c + dx]\right] - \text{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c + dx]} + b \text{Sin}[c + dx]\right]}{\left(4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4}\right) + \left(7 b (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2}\right] \text{Sin}[c + dx]^{3/2} \sqrt{1 - \text{Sin}[c + dx]^2}\right)} \right) \right) / \\
 & \left(3 \left(-7 (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2}\right] + 2 \left(2 b^2 \text{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2}\right] + (a^2 - b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2}\right] \right) \text{Sin}[c + dx]^2 \right) \right) \right) + \frac{1}{12 (a + b \text{Cos}[c + dx]) \sqrt{1 - \text{Sin}[c + dx]^2}} \\
 & (2 a^5 - 10 a^3 b^2 - 22 a b^4) \text{Cos}[c + dx] \left(a + b \sqrt{1 - \text{Sin}[c + dx]^2} \right) \\
 & \left(\left((3 + 3 i) \left(2 \text{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\text{Sin}[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \text{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\text{Sin}[c + dx]}}{(-a^2 + b^2)^{1/4}}\right] - \text{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\text{Sin}[c + dx]} + i b \text{Sin}[c + dx]\right] + \text{Log}\left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\text{Sin}[c + dx]} + i b \text{Sin}[c + dx]\right] \right) \right) / \left(\sqrt{b} (-a^2 + b^2)^{1/4} \right) + \\
 & \left(56 a (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2}\right] \text{Sin}[c + dx]^{3/2} \right) / \left(\sqrt{1 - \text{Sin}[c + dx]^2} \left(7 (a^2 - b^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \text{Sin}[c + dx]^2, \frac{b^2 \text{Sin}[c + dx]^2}{-a^2 + b^2}\right] \right) \text{Sin}[c + dx]^2 \right) \right) \right) \right)
 \end{aligned}$$

Problem 78: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \sin [c + d x])^{13/2}}{(a + b \cos [c + d x])^3} dx$$

Optimal (type 4, 590 leaves, 15 steps):

$$\frac{11 (9 a^4 - 11 a^2 b^2 + 2 b^4) e^{13/2} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{e \sin [c + d x]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{8 b^{13/2} (-a^2 + b^2)^{1/4} d} -$$

$$\frac{11 (9 a^4 - 11 a^2 b^2 + 2 b^4) e^{13/2} \operatorname{ArcTanh} \left[\frac{\sqrt{b} \sqrt{e \sin [c + d x]}}{(-a^2 + b^2)^{1/4} \sqrt{e}} \right]}{8 b^{13/2} (-a^2 + b^2)^{1/4} d} -$$

$$\left(\frac{11 a (9 a^4 - 11 a^2 b^2 + 2 b^4) e^7 \operatorname{EllipticPi} \left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x \right), 2 \right] \sqrt{\sin [c + d x]}}{8 b^7 \left(b - \sqrt{-a^2 + b^2} \right) d \sqrt{e \sin [c + d x]}} \right) -$$

$$\left(\frac{11 a (9 a^4 - 11 a^2 b^2 + 2 b^4) e^7 \operatorname{EllipticPi} \left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x \right), 2 \right] \sqrt{\sin [c + d x]}}{8 b^7 \left(b + \sqrt{-a^2 + b^2} \right) d \sqrt{e \sin [c + d x]}} \right) +$$

$$\frac{11 a (45 a^2 - 37 b^2) e^6 \operatorname{EllipticE} \left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x \right), 2 \right] \sqrt{e \sin [c + d x]}}{20 b^6 d \sqrt{\sin [c + d x]}} -$$

$$\frac{11 e^5 (5 (9 a^2 - 2 b^2) - 27 a b \cos [c + d x]) (e \sin [c + d x])^{3/2}}{60 b^5 d} +$$

$$\frac{11 e^3 (9 a + 2 b \cos [c + d x]) (e \sin [c + d x])^{7/2}}{28 b^3 d (a + b \cos [c + d x])} + \frac{e (e \sin [c + d x])^{11/2}}{2 b d (a + b \cos [c + d x])^2}$$

Result (type 6, 1324 leaves):

$$\frac{1}{40 b^5 d \sin [c + d x]^{13/2}} 11 (e \sin [c + d x])^{13/2} \left(\frac{1}{(a + b \cos [c + d x]) (1 - \sin [c + d x])^2} \right. \\ \left. 2 (45 a^3 - 37 a b^2) \cos [c + d x]^2 \left(a + b \sqrt{1 - \sin [c + d x]^2} \right) \right. \\ \left. \left(\left(a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c + d x]}}{(a^2 - b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c + d x]}}{(a^2 - b^2)^{1/4}} \right] \right) + \right. \right. \\ \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin [c + d x]} + b \sin [c + d x] \right] - \right. \right. \\ \left. \left. \operatorname{Log} \left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin [c + d x]} + b \sin [c + d x] \right] \right) \right) \Bigg/ \\ \left(4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4} \right) + \left(7 b (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin [c + d x]^2, \right. \right. \\ \left. \left. \frac{b^2 \sin [c + d x]^2}{-a^2 + b^2} \right] \sin [c + d x]^{3/2} \sqrt{1 - \sin [c + d x]^2} \right) \Bigg/$$

$$\begin{aligned}
 & \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \\
 & \quad 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \\
 & \quad \left. \left. (a^2 - b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \right) \\
 & \quad \left. \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \Bigg) + \\
 & \frac{1}{12 (a + b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} (18 a^2 b - 10 b^3) \cos[c + dx] \\
 & \quad \left(a + b \sqrt{1 - \sin[c + dx]^2} \right) \\
 & \quad \left(\left((3 + 3i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - \right. \right. \right. \\
 & \quad \left. \left. 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1+i) \sqrt{b} \right. \right. \right. \\
 & \quad \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] + \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1+i) \right. \right. \\
 & \quad \left. \left. \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] \right) \Bigg) / \left(\sqrt{b} (-a^2 + b^2)^{1/4} \right) + \\
 & \quad \left(56 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{3/2} \right) / \\
 & \quad \left(\sqrt{1 - \sin[c + dx]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 (a^2 + b^2 (-1 + \sin[c + dx]^2)) \Bigg) \Bigg) + \\
 & \frac{1}{d} \operatorname{Csc}[c + dx]^6 (e \sin[c + dx])^{13/2} \left(\frac{(-168 a^2 + 65 b^2) \sin[c + dx]}{42 b^5} - \right. \\
 & \quad \frac{19 (a^3 \sin[c + dx] - a b^2 \sin[c + dx])}{4 b^5 (a + b \cos[c + dx])} + \\
 & \quad \frac{a^4 \sin[c + dx] - 2 a^2 b^2 \sin[c + dx] + b^4 \sin[c + dx]}{2 b^5 (a + b \cos[c + dx])^2} + \\
 & \quad \left. \frac{3 a \sin[2 (c + dx)]}{5 b^4} - \right)
 \end{aligned}$$

$$\frac{\sin\left[3(c+dx)\right]}{14b^3}$$

Problem 79: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \sin[c+dx])^{11/2}}{(a+b \cos[c+dx])^3} dx$$

Optimal (type 4, 604 leaves, 15 steps):

$$\begin{aligned} & - \frac{9(7a^4 - 9a^2b^2 + 2b^4) e^{11/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8b^{11/2} (-a^2+b^2)^{3/4} d} - \\ & \frac{9(7a^4 - 9a^2b^2 + 2b^4) e^{11/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8b^{11/2} (-a^2+b^2)^{3/4} d} + \\ & \frac{3a(21a^2 - 13b^2) e^6 \operatorname{EllipticF}\left[\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{4b^6 d \sqrt{e \sin[c+dx]}} - \\ & \left(9a(7a^4 - 9a^2b^2 + 2b^4) e^6 \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2+b^2}}, \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}\right) / \\ & \left(8b^6 \left(a^2 - b(b - \sqrt{-a^2+b^2})\right) d \sqrt{e \sin[c+dx]}\right) - \\ & \left(9a(7a^4 - 9a^2b^2 + 2b^4) e^6 \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2+b^2}}, \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}\right) / \\ & \left(8b^6 \left(a^2 - b(b + \sqrt{-a^2+b^2})\right) d \sqrt{e \sin[c+dx]}\right) - \\ & \frac{3e^5(3(7a^2 - 2b^2) - 7ab \cos[c+dx]) \sqrt{e \sin[c+dx]}}{4b^5 d} + \\ & \frac{9e^3(7a + 2b \cos[c+dx]) (e \sin[c+dx])^{5/2}}{20b^3 d (a+b \cos[c+dx])} + \frac{e (e \sin[c+dx])^{9/2}}{2bd (a+b \cos[c+dx])^2} \end{aligned}$$

Result (type 6, 2224 leaves):

$$\begin{aligned} & \frac{1}{d} \left(\frac{2a \cos[c+dx]}{b^4} + \frac{(-a^2+b^2)^2}{2b^5 (a+b \cos[c+dx])^2} - \frac{17a(a^2-b^2)}{4b^5 (a+b \cos[c+dx])} - \frac{\cos[2(c+dx)]}{5b^3} \right) \\ & \operatorname{Csc}[c+dx]^5 (e \sin[c+dx])^{11/2} + \\ & \frac{1}{40b^5 d \sin[c+dx]^{11/2}} 3 (e \sin[c+dx])^{11/2} \left(\frac{1}{(a+b \cos[c+dx]) (1-\sin[c+dx])^2} \right) \end{aligned}$$

$$\begin{aligned}
 & 2 (25 a^3 - 37 a b^2) \operatorname{Cos}[c + d x]^2 \left(a + b \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) \\
 & \left(\left(a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] - \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + b \operatorname{Sin}[c + d x]\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + b \operatorname{Sin}[c + d x]\right] \right) \right) / \\
 & \left(4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4} \right) + \left(5 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \sqrt{\operatorname{Sin}[c + d x]} \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) / \\
 & \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] + \right. \right. \\
 & \quad \left. \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] + (a^2 - b^2) \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \right) \operatorname{Sin}[c + d x]^2 \right) \\
 & \left. \left(a^2 + b^2 (-1 + \operatorname{Sin}[c + d x]^2) \right) \right) + \frac{1}{(a + b \operatorname{Cos}[c + d x]) \sqrt{1 - \operatorname{Sin}[c + d x]^2}} \\
 & 2 (30 a^2 b - 16 b^3) \operatorname{Cos}[c + d x] \left(a + b \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) \left(-\frac{1}{(-a^2 + b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \right. \\
 & \left(2 \operatorname{ArcTan}\left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] - 2 \operatorname{ArcTan}\left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(-a^2 + b^2)^{1/4}}\right] \right) + \\
 & \left. \operatorname{Log}\left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i b \operatorname{Sin}[c + d x]\right] - \right. \\
 & \left. \operatorname{Log}\left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + i b \operatorname{Sin}[c + d x]\right] \right) + \\
 & \left(5 a (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \sqrt{\operatorname{Sin}[c + d x]} \right) / \\
 & \left(\sqrt{1 - \operatorname{Sin}[c + d x]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] - 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] + (-a^2 + b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c + d x]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2}\right] \right) \operatorname{Sin}[c + d x]^2 \left(a^2 + b^2 (-1 + \operatorname{Sin}[c + d x]^2) \right) \right) \right) + \\
 & \frac{1}{(a + b \operatorname{Cos}[c + d x]) (1 - 2 \operatorname{Sin}[c + d x]^2) \sqrt{1 - \operatorname{Sin}[c + d x]^2}} (-40 a^2 b + 14 b^3)
 \end{aligned}$$

$$\begin{aligned}
 & \cos [c+d x] \cos [2(c+d x)] \\
 & \left(a+b \sqrt{1-\sin [c+d x]^2} \right) \\
 & \left(\frac{\left(\frac{1}{2}-\frac{i}{2} \right) \left(-2 a^2+b^2\right) \operatorname{ArcTan}\left[1-\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]}{b^{3 / 2}\left(-a^2+b^2\right)^{3 / 4}} - \right. \\
 & \left. \frac{\left(\frac{1}{2}-\frac{i}{2} \right) \left(-2 a^2+b^2\right) \operatorname{ArcTan}\left[1+\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]}{b^{3 / 2}\left(-a^2+b^2\right)^{3 / 4}} + \left(\left(\frac{1}{4}-\frac{i}{4} \right) \left(-2 a^2+b^2\right) \right. \right. \\
 & \left. \left. \operatorname{Log}\left[\sqrt{-a^2+b^2}-\left(1+i\right) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \sqrt{\sin [c+d x]}+i b \sin [c+d x]\right] \right) \right) / \\
 & \left(b^{3 / 2}\left(-a^2+b^2\right)^{3 / 4} - \left(\left(\frac{1}{4}-\frac{i}{4} \right) \left(-2 a^2+b^2\right) \operatorname{Log}\left[\sqrt{-a^2+b^2}+\left(1+i\right) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \right. \right. \right. \\
 & \left. \left. \sqrt{\sin [c+d x]}+i b \sin [c+d x]\right] \right) \right) / \left(b^{3 / 2}\left(-a^2+b^2\right)^{3 / 4} + \frac{4 \sqrt{\sin [c+d x]}}{b} + \right. \\
 & \left. \left(10 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sqrt{\sin [c+d x]} \right) \right) / \\
 & \left(\sqrt{1-\sin [c+d x]^2} \left(5\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] - 2\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] + \left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \right) \sin [c+d x]^2 \left(a^2+b^2\left(-1+\sin [c+d x]^2\right) \right) \right) - \\
 & \left(36 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sin [c+d x]^{5 / 2} \right) / \\
 & \left(5 \sqrt{1-\sin [c+d x]^2} \left(9\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] - 2\left(2 b^2 \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] + \left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \right) \sin [c+d x]^2 \left(a^2+b^2\left(-1+\sin [c+d x]^2\right) \right) \right) \right) \right)
 \end{aligned}$$

Problem 80: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \sin[c + dx])^{9/2}}{(a + b \cos[c + dx])^3} dx$$

Optimal (type 4, 498 leaves, 14 steps):

$$\begin{aligned} & -\frac{7(5a^2 - 2b^2)e^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{e\sin[c+dx]}}{(-a^2+b^2)^{1/4}\sqrt{e}}\right]}{8b^{9/2}(-a^2+b^2)^{1/4}d} + \frac{7(5a^2 - 2b^2)e^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b}\sqrt{e\sin[c+dx]}}{(-a^2+b^2)^{1/4}\sqrt{e}}\right]}{8b^{9/2}(-a^2+b^2)^{1/4}d} + \\ & \left(\frac{7a(5a^2 - 2b^2)e^5 \operatorname{EllipticPi}\left[\frac{2b}{b - \sqrt{-a^2+b^2}}, \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{8b^5\left(b - \sqrt{-a^2+b^2}\right)d\sqrt{e\sin[c+dx]}} + \right. \\ & \left. \frac{7a(5a^2 - 2b^2)e^5 \operatorname{EllipticPi}\left[\frac{2b}{b + \sqrt{-a^2+b^2}}, \frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{8b^5\left(b + \sqrt{-a^2+b^2}\right)d\sqrt{e\sin[c+dx]}} - \frac{35a^4 \operatorname{EllipticE}\left[\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{e\sin[c+dx]}}{4b^4d\sqrt{\sin[c+dx]}} + \right. \\ & \left. \frac{7e^3(5a + 2b\cos[c+dx])(e\sin[c+dx])^{3/2}}{12b^3d(a+b\cos[c+dx])} + \frac{e(e\sin[c+dx])^{7/2}}{2bd(a+b\cos[c+dx])^2} \right) \end{aligned}$$

Result (type 6, 1231 leaves):

$$\begin{aligned} & \frac{1}{d} \operatorname{Csc}[c + dx]^4 (e \sin[c + dx])^{9/2} \\ & \left(\frac{2 \sin[c + dx]}{3b^3} + \frac{11a \sin[c + dx]}{4b^3(a + b \cos[c + dx])} + \frac{-a^2 \sin[c + dx] + b^2 \sin[c + dx]}{2b^3(a + b \cos[c + dx])^2} \right) - \\ & \frac{1}{8b^3d \sin[c + dx]^{9/2}} 7 (e \sin[c + dx])^{9/2} \\ & \left(\frac{1}{(a + b \cos[c + dx]) (1 - \sin[c + dx])^2} 10a \cos[c + dx]^2 (a + b \sqrt{1 - \sin[c + dx]^2}) \right. \\ & \left. \left(\left(a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{b}\sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{b}\sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] \right) + \right. \right. \\ & \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2}\sqrt{b}(a^2 - b^2)^{1/4}\sqrt{\sin[c+dx]} + b\sin[c+dx]\right] - \right. \right. \\ & \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2}\sqrt{b}(a^2 - b^2)^{1/4}\sqrt{\sin[c+dx]} + b\sin[c+dx]\right] \right) \right) / \\ & \left(4\sqrt{2}b^{3/2}(a^2 - b^2)^{1/4} + \left(7b(a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \right. \right. \right. \\ & \quad \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{3/2} \sqrt{1 - \sin[c + dx]^2} \right) / \\ & \left(3 \left(-7(a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \end{aligned}$$

$$\begin{aligned}
 & 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] + \right. \\
 & \quad \left. (a^2-b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] \right) \\
 & \quad \left. \sin [c+d x]^2 \left(a^2+b^2 \left(-1+\sin [c+d x]^2 \right) \right) \right) \Bigg) + \\
 & \frac{1}{6(a+b \cos [c+d x]) \sqrt{1-\sin [c+d x]^2}} b \cos [c+d x] \left(a+b \sqrt{1-\sin [c+d x]^2} \right) \\
 & \left(\left((3+3 i) \left(2 \operatorname{ArcTan} \left[1-\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right] - \right. \right. \right. \\
 & \quad \left. \left. 2 \operatorname{ArcTan} \left[1+\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2+b^2}-(1+i) \sqrt{b} \right. \right. \right. \\
 & \quad \left. \left. (-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]}+i b \sin [c+d x] \right] + \operatorname{Log} \left[\sqrt{-a^2+b^2}+(1+i) \right. \right. \\
 & \quad \left. \left. \sqrt{b}(-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]}+i b \sin [c+d x] \right] \right) \Bigg) / \left(\sqrt{b}(-a^2+b^2)^{1/4} \right) + \\
 & \left(56 a \left(a^2-b^2 \right) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] \sin [c+d x]^{3/2} \right) / \\
 & \left(\sqrt{1-\sin [c+d x]^2} \left(7 \left(a^2-b^2 \right) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin [c+d x]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] + \left(-a^2+b^2 \right) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin [c+d x]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] \right) \sin [c+d x]^2 \left(a^2+b^2 \left(-1+\sin [c+d x]^2 \right) \right) \right) \Bigg) \Bigg)
 \end{aligned}$$

Problem 81: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \sin [c+d x])^{7/2}}{(a+b \cos [c+d x])^3} dx$$

Optimal (type 4, 512 leaves, 14 steps):

$$\begin{aligned}
 & \frac{5 (3 a^2 - 2 b^2) e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{7/2} (-a^2+b^2)^{3/4} d} + \frac{5 (3 a^2 - 2 b^2) e^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin [c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{7/2} (-a^2+b^2)^{3/4} d} - \\
 & \frac{15 a e^4 \operatorname{EllipticF}\left[\frac{1}{2}\left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\sin [c+d x]}}{4 b^4 d \sqrt{e \sin [c+d x]}} + \\
 & \left(5 a (3 a^2 - 2 b^2) e^4 \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2+b^2}}, \frac{1}{2}\left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\sin [c+d x]}\right) / \\
 & \left(8 b^4 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin [c+d x]}\right) + \\
 & \left(5 a (3 a^2 - 2 b^2) e^4 \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2+b^2}}, \frac{1}{2}\left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\sin [c+d x]}\right) / \\
 & \left(8 b^4 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \sin [c+d x]}\right) + \\
 & \frac{5 e^3 (3 a + 2 b \cos [c+d x]) \sqrt{e \sin [c+d x]}}{4 b^3 d (a + b \cos [c+d x])} + \frac{e (e \sin [c+d x])^{5/2}}{2 b d (a + b \cos [c+d x])^2}
 \end{aligned}$$

Result (type 6, 2154 leaves):

$$\begin{aligned}
 & \frac{\left(\frac{-a^2+b^2}{2 b^3 (a+b \cos [c+d x])^2} + \frac{9 a}{4 b^3 (a+b \cos [c+d x])}\right) \operatorname{Csc}[c+d x]^3 (e \sin [c+d x])^{7/2}}{d} - \\
 & \frac{1}{8 b^3 d \sin [c+d x]^{7/2}} (e \sin [c+d x])^{7/2} \\
 & \left(\frac{1}{(a+b \cos [c+d x]) (1 - \sin [c+d x])^2} 14 a \cos [c+d x]^2 \left(a + b \sqrt{1 - \sin [c+d x]^2}\right) \right. \\
 & \left. \left(\left(a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c+d x]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin [c+d x]}}{(a^2 - b^2)^{1/4}}\right] \right) - \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin [c+d x]} + b \sin [c+d x]\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin [c+d x]} + b \sin [c+d x]\right] \right) \right) / \\
 & \left(4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4} \right) + \left(5 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2 + b^2}\right] \sqrt{\sin [c+d x]} \sqrt{1 - \sin [c+d x]^2} \right) / \\
 & \left(\left(-5 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2 + b^2}\right] + \right. \right. \\
 & \quad \left. \left. 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2 + b^2}\right] + (a^2 - b^2) \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2 + b^2}\right] \right) \sin [c+d x]^2 \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left(10 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sqrt{\sin[c + dx]} \right) / \\
 & \left(\sqrt{1 - \sin[c + dx]^2} \left(5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) - \\
 & \left(36 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{5/2} \right) / \\
 & \left(5 \sqrt{1 - \sin[c + dx]^2} \left(9 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \Bigg) \Bigg)
 \end{aligned}$$

Problem 82: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \sin[c + dx])^{5/2}}{(a + b \cos[c + dx])^3} dx$$

Optimal (type 4, 520 leaves, 14 steps):

$$\begin{aligned}
& - \frac{3 (a^2 - 2 b^2) e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{5/2} (-a^2+b^2)^{5/4} d} + \frac{3 (a^2 - 2 b^2) e^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{5/2} (-a^2+b^2)^{5/4} d} \\
& \left(\frac{3 a (a^2 - 2 b^2) e^3 \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{8 b^3 (a^2 - b^2) \left(b - \sqrt{-a^2+b^2}\right) d \sqrt{e \sin[c+dx]}} - \right. \\
& \left. \frac{3 a (a^2 - 2 b^2) e^3 \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c+dx]}}{8 b^3 (a^2 - b^2) \left(b + \sqrt{-a^2+b^2}\right) d \sqrt{e \sin[c+dx]}} + \right. \\
& \left. \frac{3 a e^2 \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{e \sin[c+dx]}}{4 b^2 (a^2 - b^2) d \sqrt{\sin[c+dx]}} + \right. \\
& \left. \frac{e (e \sin[c+dx])^{3/2}}{2 b d (a + b \cos[c+dx])^2} - \frac{3 a e (e \sin[c+dx])^{3/2}}{4 b (a^2 - b^2) d (a + b \cos[c+dx])} \right)
\end{aligned}$$

Result (type 6, 1225 leaves):

$$\begin{aligned}
& \frac{1}{d} \operatorname{Csc}[c+dx]^2 (e \sin[c+dx])^{5/2} \left(\frac{\sin[c+dx]}{2 b (a + b \cos[c+dx])^2} + \frac{3 a \sin[c+dx]}{4 b (-a^2+b^2) (a + b \cos[c+dx])} \right) + \\
& \frac{1}{8 (a-b) b (a+b) d \sin[c+dx]^{5/2}} 3 (e \sin[c+dx])^{5/2} \\
& \left(\frac{1}{(a + b \cos[c+dx]) (1 - \sin[c+dx])^2} 2 a \cos[c+dx]^2 \left(a + b \sqrt{1 - \sin[c+dx]^2} \right) \right. \\
& \left. \left(\left(a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2 - b^2)^{1/4}}\right] \right) + \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] - \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx]\right] \right) \right) / \\
& \left(4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4} + \left(7 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \sin[c+dx]^{3/2} \sqrt{1 - \sin[c+dx]^2} \right) / \\
& \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + \right. \right. \\
& \quad \left. \left. 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned} & \left. \left. \left. \left. \left. \sin [c+d x]^2 \left(a^2+b^2(-1+\sin [c+d x]^2)\right)\right)\right)\right)\right) + \\ & \frac{1}{6(a+b \cos [c+d x]) \sqrt{1-\sin [c+d x]^2}} b \cos [c+d x] \left(a+b \sqrt{1-\sin [c+d x]^2}\right) \\ & \left(\left((3+3 i) \left(2 \operatorname{ArcTan} \left[1-\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right] - \right. \right. \right. \\ & \quad \left. \left. \left. 2 \operatorname{ArcTan} \left[1+\frac{(1+i) \sqrt{b} \sqrt{\sin [c+d x]}}{(-a^2+b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \left. (-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]} + i b \sin [c+d x] \right] + \operatorname{Log} \left[\sqrt{-a^2+b^2} + (1+i) \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \left. \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin [c+d x]} + i b \sin [c+d x] \right] \right) \right) \right) \right) / \left(\sqrt{b} (-a^2+b^2)^{1/4} \right) + \\ & \left(56 a \left(a^2-b^2\right) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] \sin [c+d x]^{3/2} \right) / \\ & \left(\sqrt{1-\sin [c+d x]^2} \left(7 \left(a^2-b^2\right) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin [c+d x]^2, \right. \right. \right. \\ & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin [c+d x]^2, \right. \right. \right. \\ & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] + \left(-a^2+b^2\right) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin [c+d x]^2, \right. \right. \right. \\ & \quad \left. \left. \frac{b^2 \sin [c+d x]^2}{-a^2+b^2} \right] \right) \sin [c+d x]^2 \left(a^2+b^2(-1+\sin [c+d x]^2)\right) \right) \right) \right) \end{aligned}$$

Problem 83: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(e \sin [c+d x])^{3/2}}{(a+b \cos [c+d x])^3} dx$$

Optimal (type 4, 534 leaves, 14 steps):

$$\begin{aligned}
 & - \frac{(a^2 + 2 b^2) e^{3/2} \text{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \text{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{3/2} (-a^2 + b^2)^{7/4} d} - \\
 & \frac{(a^2 + 2 b^2) e^{3/2} \text{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \text{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 b^{3/2} (-a^2 + b^2)^{7/4} d} - \frac{a e^2 \text{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\text{Sin}[c+dx]}}{4 b^2 (a^2 - b^2) d \sqrt{e \text{Sin}[c+dx]}} + \\
 & \left(a (a^2 + 2 b^2) e^2 \text{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\text{Sin}[c+dx]}\right) / \\
 & \left(8 b^2 (a^2 - b^2) \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \text{Sin}[c+dx]}\right) + \\
 & \left(a (a^2 + 2 b^2) e^2 \text{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\text{Sin}[c+dx]}\right) / \\
 & \left(8 b^2 (a^2 - b^2) \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) d \sqrt{e \text{Sin}[c+dx]}\right) + \\
 & \frac{e \sqrt{e \text{Sin}[c+dx]}}{2 b d (a + b \text{Cos}[c+dx])^2} - \frac{a e \sqrt{e \text{Sin}[c+dx]}}{4 b (a^2 - b^2) d (a + b \text{Cos}[c+dx])}
 \end{aligned}$$

Result (type 6, 1211 leaves):

$$\begin{aligned}
 & \frac{1}{d} \left(\frac{1}{2 b (a + b \text{Cos}[c+dx])^2} + \frac{a}{4 b (-a^2 + b^2) (a + b \text{Cos}[c+dx])} \right) \text{Csc}[c+dx] (e \text{Sin}[c+dx])^{3/2} - \\
 & \frac{1}{8 (a - b) b (a + b) d \text{Sin}[c+dx]^{3/2}} (e \text{Sin}[c+dx])^{3/2} \\
 & \left(\frac{1}{(a + b \text{Cos}[c+dx]) (1 - \text{Sin}[c+dx])^2} 2 a \text{Cos}[c+dx]^2 \left(a + b \sqrt{1 - \text{Sin}[c+dx]^2}\right) \right. \\
 & \left. \left(\left(a \left(-2 \text{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c+dx]}}{(a^2 - b^2)^{1/4}}\right] + 2 \text{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\text{Sin}[c+dx]}}{(a^2 - b^2)^{1/4}}\right] \right) - \right. \right. \\
 & \quad \left. \left. \text{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c+dx]} + b \text{Sin}[c+dx]\right] + \right. \right. \\
 & \quad \left. \left. \text{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\text{Sin}[c+dx]} + b \text{Sin}[c+dx]\right] \right) \right) / \\
 & \left(4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4} \right) + \left(5 b (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \text{Sin}[c+dx]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \text{Sin}[c+dx]^2}{-a^2 + b^2} \right] \sqrt{\text{Sin}[c+dx]} \sqrt{1 - \text{Sin}[c+dx]^2} \right) / \\
 & \left(\left(-5 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2 + b^2} \right] + \right. \right. \\
 & \quad \left. \left. 2 \left(2 b^2 \text{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \right. \right. \right. \\
 & \quad \left. \left. \left. \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \text{Sin}[c+dx]^2, \frac{b^2 \text{Sin}[c+dx]^2}{-a^2 + b^2} \right] \right) \text{Sin}[c+dx]^2 \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{(3 a^2 + 2 b^2) \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 \sqrt{b} (-a^2+b^2)^{9/4} d} + \frac{(3 a^2 + 2 b^2) \sqrt{e} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 \sqrt{b} (-a^2+b^2)^{9/4} d} + \\
 & \left(a (3 a^2 + 2 b^2) e \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\operatorname{Sin}[c + d x]}\right) / \\
 & \left(8 b (a^2 - b^2)^2 (b - \sqrt{-a^2 + b^2}) d \sqrt{e \operatorname{Sin}[c + d x]}\right) + \\
 & \left(a (3 a^2 + 2 b^2) e \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\operatorname{Sin}[c + d x]}\right) / \\
 & \left(8 b (a^2 - b^2)^2 (b + \sqrt{-a^2 + b^2}) d \sqrt{e \operatorname{Sin}[c + d x]}\right) + \\
 & \frac{5 a \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{e \operatorname{Sin}[c + d x]}}{4 (a^2 - b^2)^2 d \sqrt{\operatorname{Sin}[c + d x]}} - \\
 & \frac{b (e \operatorname{Sin}[c + d x])^{3/2}}{2 (a^2 - b^2) d e (a + b \operatorname{Cos}[c + d x])^2} - \frac{5 a b (e \operatorname{Sin}[c + d x])^{3/2}}{4 (a^2 - b^2)^2 d e (a + b \operatorname{Cos}[c + d x])}
 \end{aligned}$$

Result (type 6, 1232 leaves):

$$\begin{aligned}
 & \frac{\sqrt{e \operatorname{Sin}[c + d x]} \left(-\frac{b \operatorname{Sin}[c+d x]}{2 (a^2-b^2) (a+b \operatorname{Cos}[c+d x])^2} - \frac{5 a b \operatorname{Sin}[c+d x]}{4 (a^2-b^2)^2 (a+b \operatorname{Cos}[c+d x])} \right)}{d} + \\
 & \frac{1}{8 (a-b)^2 (a+b)^2 d \sqrt{\operatorname{Sin}[c + d x]}} \sqrt{e \operatorname{Sin}[c + d x]} \\
 & \left(\frac{1}{(a + b \operatorname{Cos}[c + d x]) (1 - \operatorname{Sin}[c + d x])^2} 10 a b \operatorname{Cos}[c + d x]^2 \left(a + b \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) \right. \\
 & \left. \left(\left(a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] \right) + \right. \right. \\
 & \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + b \operatorname{Sin}[c + d x]\right] - \right. \right. \\
 & \left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + b \operatorname{Sin}[c + d x]\right] \right) \right) / \\
 & \left(4 \sqrt{2} b^{3/2} (a^2 - b^2)^{1/4} \right) + \left(7 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \right. \right. \\
 & \left. \left. \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2} \right] \operatorname{Sin}[c + d x]^{3/2} \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) / \\
 & \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2} \right] + \right. \right. \\
 & \left. \left. 2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2} \right] + \right. \right. \right. \\
 & \left. \left. \left. (a^2 - b^2) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \operatorname{Sin}[c + d x]^2, \frac{b^2 \operatorname{Sin}[c + d x]^2}{-a^2 + b^2} \right] \right) \right)
 \end{aligned}$$

$$\left. \left. \left. \left. \left. \sin[c + dx]^2 \left(a^2 + b^2 (-1 + \sin[c + dx]^2) \right) \right) \right) \right) \right) + \frac{1}{12 (a + b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} (8a^2 + 2b^2) \cos[c + dx] \left(a + b \sqrt{1 - \sin[c + dx]^2} \right) \left(\left((3 + 3i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + ib \sin[c + dx] \right] + \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + ib \sin[c + dx] \right] \right) \right) / \left(\sqrt{b} (-a^2 + b^2)^{1/4} \right) + \left(56 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{3/2} \right) / \left(\sqrt{1 - \sin[c + dx]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \right) \sin[c + dx]^2 \right) \left(a^2 + b^2 (-1 + \sin[c + dx]^2) \right) \right) \right) \right)$$

Problem 85: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cos[c + dx])^3 \sqrt{e \sin[c + dx]}} dx$$

Optimal (type 4, 535 leaves, 14 steps):

$$\begin{aligned}
& \frac{3 \sqrt{b} (5 a^2 + 2 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{11/4} d \sqrt{e}} + \\
& \frac{3 \sqrt{b} (5 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{11/4} d \sqrt{e}} - \frac{7 a \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\operatorname{Sin}[c+d x]}}{4 (a^2-b^2)^2 d \sqrt{e \operatorname{Sin}[c+d x]}} + \\
& \left(3 a (5 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\operatorname{Sin}[c+d x]}\right) / \\
& \left(8 (a^2-b^2)^2 \left(a^2 - b \left(b - \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \operatorname{Sin}[c+d x]}\right) + \\
& \left(3 a (5 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2+b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\operatorname{Sin}[c+d x]}\right) / \\
& \left(8 (a^2-b^2)^2 \left(a^2 - b \left(b + \sqrt{-a^2+b^2}\right)\right) d \sqrt{e \operatorname{Sin}[c+d x]}\right) - \\
& \frac{b \sqrt{e \operatorname{Sin}[c+d x]}}{2 (a^2-b^2) d e (a+b \operatorname{Cos}[c+d x])^2} - \frac{7 a b \sqrt{e \operatorname{Sin}[c+d x]}}{4 (a^2-b^2)^2 d e (a+b \operatorname{Cos}[c+d x])}
\end{aligned}$$

Result (type 6, 1226 leaves):

$$\begin{aligned}
& \frac{\left(-\frac{b}{2 (a^2-b^2) (a+b \operatorname{Cos}[c+d x])^2} - \frac{7 a b}{4 (a^2-b^2)^2 (a+b \operatorname{Cos}[c+d x])}\right) \operatorname{Sin}[c+d x]}{d \sqrt{e \operatorname{Sin}[c+d x]}} + \\
& \frac{1}{8 (a-b)^2 (a+b)^2 d \sqrt{e \operatorname{Sin}[c+d x]}} \sqrt{\operatorname{Sin}[c+d x]} \\
& \left(-\frac{1}{(a+b \operatorname{Cos}[c+d x]) (1-\operatorname{Sin}[c+d x])^2} 14 a b \operatorname{Cos}[c+d x]^2 \left(a+b \sqrt{1-\operatorname{Sin}[c+d x]^2}\right)\right. \\
& \left.\left(\left(a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+d x]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+d x]}}{(a^2-b^2)^{1/4}}\right]\right) - \operatorname{Log}\left[\frac{\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + b \operatorname{Sin}[c+d x]}{\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + b \operatorname{Sin}[c+d x]}\right]\right)\right) / \\
& \left(4 \sqrt{2} \sqrt{b} (a^2-b^2)^{3/4} + \left(5 b (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \right.\right.\right. \\
& \left.\left.\left.\operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2}\right] \sqrt{\operatorname{Sin}[c+d x]} \sqrt{1-\operatorname{Sin}[c+d x]^2}\right)\right) / \\
& \left(\left(-5 (a^2-b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2}\right] + 2\right.\right. \\
& \left.\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2}\right] +\right.\right. \\
& \left.\left.(a^2-b^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \operatorname{Sin}[c+d x]^2, \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2}\right]\right)\right)
\end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \left. \left. \sin [c+d x]^2\right)\left(a^2+b^2\left(-1+\sin [c+d x]^2\right)\right)\right)\right)\right)\right) + \\
 & \frac{1}{(a+b \cos [c+d x]) \sqrt{1-\sin [c+d x]^2}} 2\left(8 a^2+6 b^2\right) \cos [c+d x] \\
 & \left(a+b \sqrt{1-\sin [c+d x]^2}\right) \\
 & \left(-\frac{1}{\left(-a^2+b^2\right)^{3 / 4}}\left(\frac{1}{8}-\frac{i}{8}\right) \sqrt{b}\right. \\
 & \left. \left(2 \operatorname{ArcTan}\left[1-\frac{\left(1+i\right) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]-2 \operatorname{ArcTan}\left[1+\frac{\left(1+i\right) \sqrt{b} \sqrt{\sin [c+d x]}}{\left(-a^2+b^2\right)^{1 / 4}}\right]\right)+ \\
 & \log \left[\sqrt{-a^2+b^2}-\left(1+i\right) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]- \\
 & \log \left[\sqrt{-a^2+b^2}+\left(1+i\right) \sqrt{b}\left(-a^2+b^2\right)^{1 / 4} \sqrt{\sin [c+d x]}+i b \sin [c+d x]\right]\right) + \\
 & \left(5 a\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right] \sqrt{\sin [c+d x]}\right) / \\
 & \left(\sqrt{1-\sin [c+d x]^2}\left(5\left(a^2-b^2\right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]-2\left(2 b^2 \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]\right.\right.\right. \\
 & \left.\left.\left.+\left(-a^2+b^2\right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin [c+d x]^2, \frac{b^2 \sin [c+d x]^2}{-a^2+b^2}\right]\right)\right) \sin [c+d x]^2\right)\left(a^2+b^2\left(-1+\sin [c+d x]^2\right)\right)\right)\right)\right)
 \end{aligned}$$

Problem 86: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+b \cos [c+d x])^3 (e \sin [c+d x])^{3 / 2}} d x$$

Optimal (type 4, 611 leaves, 15 steps):

$$\begin{aligned}
& - \frac{5 b^{3/2} (7 a^2 + 2 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{13/4} d e^{3/2}} + \frac{5 b^{3/2} (7 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+d x]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{13/4} d e^{3/2}} \\
& - \frac{b}{2 (a^2-b^2) d e (a+b \operatorname{Cos}[c+d x])^2 \sqrt{e \operatorname{Sin}[c+d x]}} \\
& + \frac{9 a b}{4 (a^2-b^2)^2 d e (a+b \operatorname{Cos}[c+d x]) \sqrt{e \operatorname{Sin}[c+d x]}} + \frac{5 b (7 a^2+2 b^2) - a (8 a^2+37 b^2) \operatorname{Cos}[c+d x]}{4 (a^2-b^2)^3 d e \sqrt{e \operatorname{Sin}[c+d x]}} \\
& \left(\frac{5 a b (7 a^2+2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b-\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\operatorname{Sin}[c+d x]}}{8 (a^2-b^2)^3 (b-\sqrt{-a^2+b^2}) d e \sqrt{e \operatorname{Sin}[c+d x]}} \right) / \\
& - \left(\frac{5 a b (7 a^2+2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b+\sqrt{-a^2+b^2}}, \frac{1}{2} \left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{\operatorname{Sin}[c+d x]}}{8 (a^2-b^2)^3 (b+\sqrt{-a^2+b^2}) d e \sqrt{e \operatorname{Sin}[c+d x]}} \right) / \\
& - \frac{a (8 a^2+37 b^2) \operatorname{EllipticE}\left[\frac{1}{2} \left(c-\frac{\pi}{2}+d x\right), 2\right] \sqrt{e \operatorname{Sin}[c+d x]}}{4 (a^2-b^2)^3 d e^2 \sqrt{\operatorname{Sin}[c+d x]}}
\end{aligned}$$

Result (type 6, 1316 leaves):

$$\begin{aligned}
& \left(\operatorname{Sin}[c+d x]^2 \left(-\frac{1}{(a^2-b^2)^3} 2 (-3 a^2 b - b^3 + a^3 \operatorname{Cos}[c+d x] + 3 a b^2 \operatorname{Cos}[c+d x]) \operatorname{Csc}[c+d x] + \right. \right. \\
& \left. \left. \frac{b^3 \operatorname{Sin}[c+d x]}{2 (a^2-b^2)^2 (a+b \operatorname{Cos}[c+d x])^2} + \frac{13 a b^3 \operatorname{Sin}[c+d x]}{4 (a^2-b^2)^3 (a+b \operatorname{Cos}[c+d x])} \right) \right) / \\
& \left(d (e \operatorname{Sin}[c+d x])^{3/2} \right) - \frac{1}{8 (a-b)^3 (a+b)^3 d (e \operatorname{Sin}[c+d x])^{3/2}} \\
& \operatorname{Sin}[c+d x]^{3/2} \left(\frac{1}{(a+b \operatorname{Cos}[c+d x]) (1-\operatorname{Sin}[c+d x])^2} \right. \\
& \left. 2 (8 a^3 b + 37 a b^3) \operatorname{Cos}[c+d x]^2 (a+b \sqrt{1-\operatorname{Sin}[c+d x]^2}) \right. \\
& \left. \left(\left(a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+d x]}}{(a^2-b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c+d x]}}{(a^2-b^2)^{1/4}}\right] \right) + \right. \right. \\
& \left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + b \operatorname{Sin}[c+d x]\right] - \right. \right. \\
& \left. \left. \operatorname{Log}\left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\operatorname{Sin}[c+d x]} + b \operatorname{Sin}[c+d x]\right] \right) \right) / \\
& \left(4 \sqrt{2} b^{3/2} (a^2-b^2)^{1/4} \right) + \left(7 b (a^2-b^2) \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \operatorname{Sin}[c+d x]^2, \right. \right. \\
& \left. \left. \frac{b^2 \operatorname{Sin}[c+d x]^2}{-a^2+b^2} \right] \operatorname{Sin}[c+d x]^{3/2} \sqrt{1-\operatorname{Sin}[c+d x]^2} \right) /
\end{aligned}$$

$$\begin{aligned}
 & \left(3 \left(-7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + \right. \right. \\
 & \quad 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (a^2 - b^2) \right. \\
 & \quad \quad \left. \left. \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) \\
 & \quad \left. \left. \left. (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) \right) + \frac{1}{12 (a + b \cos[c + dx]) \sqrt{1 - \sin[c + dx]^2}} \\
 & (8 a^4 + 72 a^2 b^2 + 10 b^4) \cos[c + dx] \left(a + b \sqrt{1 - \sin[c + dx]^2} \right) \\
 & \left(\left((3 + 3 i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1 + i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - \right. \right. \right. \\
 & \quad 2 \operatorname{ArcTan} \left[1 + \frac{(1 + i) \sqrt{b} \sqrt{\sin[c + dx]}}{(-a^2 + b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2 + b^2} - (1 + i) \sqrt{b} \right. \\
 & \quad \left. \left. (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] + \operatorname{Log} \left[\sqrt{-a^2 + b^2} + (1 + i) \right. \right. \\
 & \quad \left. \left. \left. \sqrt{b} (-a^2 + b^2)^{1/4} \sqrt{\sin[c + dx]} + i b \sin[c + dx] \right] \right) \right) \left. \right) / \left(\sqrt{b} (-a^2 + b^2)^{1/4} \right) + \\
 & \left(56 a (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \sin[c + dx]^{3/2} \right) / \\
 & \left(\sqrt{1 - \sin[c + dx]^2} \left(7 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c + dx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c + dx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] + (-a^2 + b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c + dx]^2, \right. \right. \\
 & \quad \left. \left. \left. \frac{b^2 \sin[c + dx]^2}{-a^2 + b^2} \right] \right) \sin[c + dx]^2 \right) (a^2 + b^2 (-1 + \sin[c + dx]^2)) \right) \right) \right)
 \end{aligned}$$

Problem 87: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cos[c + dx])^3 (e \sin[c + dx])^{5/2}} dx$$

Optimal (type 4, 629 leaves, 15 steps):

$$\frac{7 b^{5/2} (9 a^2 + 2 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{15/4} d e^{5/2}} + \frac{7 b^{5/2} (9 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Sin}[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{15/4} d e^{5/2}} -$$

$$\frac{b}{2 (a^2 - b^2) d e (a + b \operatorname{Cos}[c + d x])^2 (e \operatorname{Sin}[c + d x])^{3/2}} -$$

$$\frac{11 a b}{4 (a^2 - b^2)^2 d e (a + b \operatorname{Cos}[c + d x]) (e \operatorname{Sin}[c + d x])^{3/2}} +$$

$$\frac{7 b (9 a^2 + 2 b^2) - a (8 a^2 + 69 b^2) \operatorname{Cos}[c + d x]}{12 (a^2 - b^2)^3 d e (e \operatorname{Sin}[c + d x])^{3/2}} +$$

$$\frac{a (8 a^2 + 69 b^2) \operatorname{EllipticF}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\operatorname{Sin}[c + d x]}}{12 (a^2 - b^2)^3 d e^2 \sqrt{e \operatorname{Sin}[c + d x]}} -$$

$$\left(\frac{7 a b^2 (9 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\operatorname{Sin}[c + d x]}}{8 (a^2 - b^2)^3 \left(a^2 - b \left(b - \sqrt{-a^2 + b^2}\right)\right) d e^2 \sqrt{e \operatorname{Sin}[c + d x]}} \right) /$$

$$\left(\frac{7 a b^2 (9 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + d x\right), 2\right] \sqrt{\operatorname{Sin}[c + d x]}}{8 (a^2 - b^2)^3 \left(a^2 - b \left(b + \sqrt{-a^2 + b^2}\right)\right) d e^2 \sqrt{e \operatorname{Sin}[c + d x]}} \right) /$$

Result(type 6, 1308 leaves):

$$\left(\left(\frac{b^3}{2 (a^2 - b^2)^2 (a + b \operatorname{Cos}[c + d x])^2} + \frac{15 a b^3}{4 (a^2 - b^2)^3 (a + b \operatorname{Cos}[c + d x])} - \frac{1}{3 (a^2 - b^2)^3} \right. \right.$$

$$\left. \left. 2 (-3 a^2 b - b^3 + a^3 \operatorname{Cos}[c + d x] + 3 a b^2 \operatorname{Cos}[c + d x]) \operatorname{Csc}[c + d x]^2 \right) \operatorname{Sin}[c + d x]^3 \right) /$$

$$\left(d (e \operatorname{Sin}[c + d x])^{5/2} \right) + \frac{1}{24 (a - b)^3 (a + b)^3 d (e \operatorname{Sin}[c + d x])^{5/2}}$$

$$\operatorname{Sin}[c + d x]^{5/2} \left(\frac{1}{(a + b \operatorname{Cos}[c + d x]) (1 - \operatorname{Sin}[c + d x])^2} \right.$$

$$\left. 2 (8 a^3 b + 69 a b^3) \operatorname{Cos}[c + d x]^2 \left(a + b \sqrt{1 - \operatorname{Sin}[c + d x]^2} \right) \right.$$

$$\left(\left(a \left(-2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] + 2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\operatorname{Sin}[c + d x]}}{(a^2 - b^2)^{1/4}}\right] \right) - \right.$$

$$\left. \operatorname{Log}\left[\sqrt{a^2 - b^2} - \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + b \operatorname{Sin}[c + d x]\right] + \right.$$

$$\left. \left. \operatorname{Log}\left[\sqrt{a^2 - b^2} + \sqrt{2} \sqrt{b} (a^2 - b^2)^{1/4} \sqrt{\operatorname{Sin}[c + d x]} + b \operatorname{Sin}[c + d x]\right] \right) \right) /$$

$$\left(4 \sqrt{2} \sqrt{b} (a^2 - b^2)^{3/4} \right) + \left(5 b (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \operatorname{Sin}[c + d x]^2, \right. \right.$$

$$\begin{aligned}
 & \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \sqrt{\sin[c+dx]} \sqrt{1-\sin[c+dx]^2} \right) / \\
 & \left(\left(-5 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{4}, -\frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + \right. \right. \\
 & \quad 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + (a^2-b^2) \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \right) \sin[c+dx]^2 \right) \\
 & \left. (a^2+b^2(-1+\sin[c+dx]^2)) \right) \Bigg) + \frac{1}{(a+b \cos[c+dx]) \sqrt{1-\sin[c+dx]^2}} \\
 & 2 (8 a^4 - 120 a^2 b^2 - 42 b^4) \cos[c+dx] \left(a + b \sqrt{1-\sin[c+dx]^2} \right) \\
 & \left(-\frac{1}{(-a^2+b^2)^{3/4}} \left(\frac{1}{8} - \frac{i}{8} \right) \sqrt{b} \right. \\
 & \quad \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}} \right] - 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}} \right] \right) + \\
 & \quad \operatorname{Log} \left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx] \right] - \\
 & \quad \left. \operatorname{Log} \left[\sqrt{-a^2+b^2} + (1+i) \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx] \right] \right) + \\
 & \left(5 a (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \sqrt{\sin[c+dx]} \right) / \\
 & \left(\sqrt{1-\sin[c+dx]^2} \left(5 (a^2-b^2) \operatorname{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \sin[c+dx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \sin[c+dx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \sin[c+dx]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \right) \sin[c+dx]^2 (a^2+b^2(-1+\sin[c+dx]^2)) \Bigg) \Bigg)
 \end{aligned}$$

Problem 88: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+b \cos[c+dx])^3 (e \sin[c+dx])^{7/2}} dx$$

Optimal (type 4, 700 leaves, 16 steps):

$$\begin{aligned}
& - \frac{9 b^{7/2} (11 a^2 + 2 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{17/4} d e^{7/2}} + \frac{9 b^{7/2} (11 a^2 + 2 b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{b} \sqrt{e \sin[c+dx]}}{(-a^2+b^2)^{1/4} \sqrt{e}}\right]}{8 (-a^2+b^2)^{17/4} d e^{7/2}} \\
& - \frac{b}{2 (a^2 - b^2) d e (a + b \cos[c + dx])^2 (e \sin[c + dx])^{5/2}} \\
& + \frac{13 a b}{4 (a^2 - b^2)^2 d e (a + b \cos[c + dx]) (e \sin[c + dx])^{5/2}} \\
& - \frac{9 b (11 a^2 + 2 b^2) - a (8 a^2 + 109 b^2) \cos[c + dx]}{20 (a^2 - b^2)^3 d e (e \sin[c + dx])^{5/2}} \\
& + \frac{3 (15 b^3 (11 a^2 + 2 b^2) + a (8 a^4 - 64 a^2 b^2 - 139 b^4) \cos[c + dx])}{20 (a^2 - b^2)^4 d e^3 \sqrt{e \sin[c + dx]}} \\
& \left(9 a b^3 (11 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b - \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}\right) / \\
& \left(8 (a^2 - b^2)^4 (b - \sqrt{-a^2 + b^2}) d e^3 \sqrt{e \sin[c + dx]}\right) + \\
& \left(9 a b^3 (11 a^2 + 2 b^2) \operatorname{EllipticPi}\left[\frac{2 b}{b + \sqrt{-a^2 + b^2}}, \frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{\sin[c + dx]}\right) / \\
& \left(8 (a^2 - b^2)^4 (b + \sqrt{-a^2 + b^2}) d e^3 \sqrt{e \sin[c + dx]}\right) - \\
& \left(3 a (8 a^4 - 64 a^2 b^2 - 139 b^4) \operatorname{EllipticE}\left[\frac{1}{2} \left(c - \frac{\pi}{2} + dx\right), 2\right] \sqrt{e \sin[c + dx]}\right) / \\
& \left(20 (a^2 - b^2)^4 d e^4 \sqrt{\sin[c + dx]}\right)
\end{aligned}$$

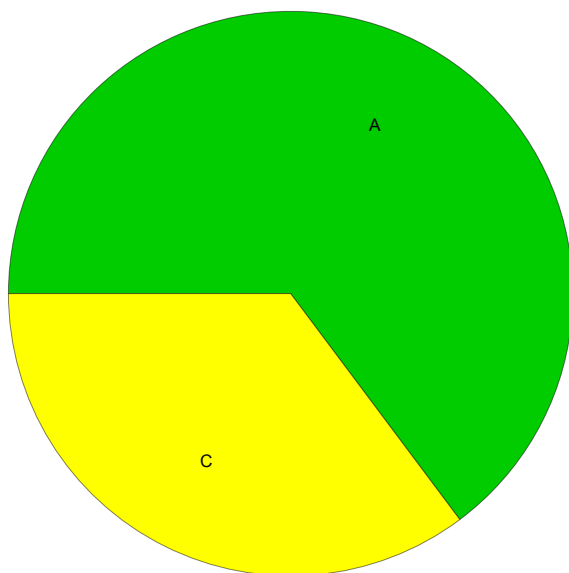
Result (type 6, 1408 leaves):

$$\begin{aligned}
& \left(\sin[c + dx]^4 \right. \\
& \left(- \frac{1}{5 (a^2 - b^2)^4} 2 (50 a^2 b^3 + 10 b^5 + 3 a^5 \cos[c + dx] - 24 a^3 b^2 \cos[c + dx] - 39 a b^4 \cos[c + dx]) \right. \\
& \left. \operatorname{Csc}[c + dx] - \frac{1}{5 (a^2 - b^2)^3} 2 (-3 a^2 b - b^3 + a^3 \cos[c + dx] + 3 a b^2 \cos[c + dx]) \right. \\
& \left. \operatorname{Csc}[c + dx]^3 - \frac{b^5 \sin[c + dx]}{2 (a^2 - b^2)^3 (a + b \cos[c + dx])^2} - \frac{21 a b^5 \sin[c + dx]}{4 (a^2 - b^2)^4 (a + b \cos[c + dx])} \right) / \\
& \left(d (e \sin[c + dx])^{7/2} \right) - \frac{1}{40 (a - b)^4 (a + b)^4 d (e \sin[c + dx])^{7/2}} \\
& 3 \sin[c + dx]^{7/2} \left(\frac{1}{(a + b \cos[c + dx]) (1 - \sin[c + dx])^2} \right. \\
& \left. 2 (8 a^5 b - 64 a^3 b^3 - 139 a b^5) \cos[c + dx]^2 (a + b \sqrt{1 - \sin[c + dx]})^2 \right)
\end{aligned}$$

$$\begin{aligned}
 & \left(\left(a \left(-2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}} \right] + 2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} \sqrt{b} \sqrt{\sin[c+dx]}}{(a^2-b^2)^{1/4}} \right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Log} \left[\sqrt{a^2-b^2} - \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx] \right] - \right. \right. \\
 & \quad \left. \left. \operatorname{Log} \left[\sqrt{a^2-b^2} + \sqrt{2} \sqrt{b} (a^2-b^2)^{1/4} \sqrt{\sin[c+dx]} + b \sin[c+dx] \right] \right) \right) / \\
 & \quad \left(4 \sqrt{2} b^{3/2} (a^2-b^2)^{1/4} + \left(7 b (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \sin[c+dx]^{3/2} \sqrt{1-\sin[c+dx]^2} \right) / \\
 & \quad \left(3 \left(-7 (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, -\frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + \right. \right. \\
 & \quad \left. \left. 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, -\frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + (a^2-b^2) \right. \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \right) \sin[c+dx]^2 \right) \\
 & \quad \left. \left. \left. (a^2+b^2 (-1+\sin[c+dx]^2)) \right) \right) \right) + \frac{1}{12 (a+b \cos[c+dx]) \sqrt{1-\sin[c+dx]^2}} \\
 & (8 a^6 - 64 a^4 b^2 - 304 a^2 b^4 - 30 b^6) \cos[c+dx] \left(a + b \sqrt{1-\sin[c+dx]^2} \right) \\
 & \left(\left((3+3i) \left(2 \operatorname{ArcTan} \left[1 - \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}} \right] - \right. \right. \right. \\
 & \quad \left. \left. 2 \operatorname{ArcTan} \left[1 + \frac{(1+i) \sqrt{b} \sqrt{\sin[c+dx]}}{(-a^2+b^2)^{1/4}} \right] - \operatorname{Log} \left[\sqrt{-a^2+b^2} - (1+i) \sqrt{b} \right. \right. \right. \\
 & \quad \left. \left. (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx] \right] + \operatorname{Log} \left[\sqrt{-a^2+b^2} + (1+i) \right. \right. \\
 & \quad \left. \left. \sqrt{b} (-a^2+b^2)^{1/4} \sqrt{\sin[c+dx]} + i b \sin[c+dx] \right] \right) \right) / \left(\sqrt{b} (-a^2+b^2)^{1/4} + \right. \\
 & \quad \left. \left(56 a (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \sin[c+dx]^{3/2} \right) / \right. \\
 & \quad \left(\sqrt{1-\sin[c+dx]^2} \left(7 (a^2-b^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, \sin[c+dx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] - 2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, 2, \frac{11}{4}, \sin[c+dx]^2, \right. \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] + (-a^2+b^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, 1, \frac{11}{4}, \sin[c+dx]^2, \right. \right. \\
 & \quad \left. \left. \frac{b^2 \sin[c+dx]^2}{-a^2+b^2} \right] \right) \sin[c+dx]^2 \left. \left. \left. (a^2+b^2 (-1+\sin[c+dx]^2)) \right) \right) \right) \right) \right)
 \end{aligned}$$

Summary of Integration Test Results

88 integration problems



A - 57 optimal antiderivatives

B - 0 more than twice size of optimal antiderivatives

C - 31 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts